$$\begin{tikzcd} A \arrow[r, "H", "|" marking] & B \arrow[r, "K", "|" marking] & C \arrow[r, "L", "|" marking] & D \end{tikzcd}$$

$$\begin{tikzcd}
\bullet \arrow[r, "f"] \arrow[d] & \bullet \arrow[rr, "k"] \arrow[d] & & \bullet \arrow[r, "n"] \arrow[d] \arrow[dr, phantom, "\Downarrow\varphi"] & \bullet \arrow[dd] \\
\bullet \arrow[r, "g"] \arrow[rr, bend right, "g'"'] \arrow[d] \arrow[u, phantom, "\Downarrow\alpha"] \arrow[dr, phantom, "\Downarrow\beta"] & \bullet \arrow[r, "b"] \arrow[rr, bend left, "\ell"] \arrow[urr, phantom, "\Downarrow\gamma"] & \bullet \arrow[r, "\ell'"] \arrow[d] \arrow[dr, phantom, "\Downarrow\delta"] & \bullet \arrow[d] & \\
\bullet \arrow[rr, "h"] & & \bullet \arrow[r, "m"] & \bullet \arrow[r, "p"] & \bullet
\end{tikzcd}$$

α & β can't be composed in H

γ & δ can't be composed in K

But in $H \odot K$, (α,γ) has vertical codomain (g,ℓ)
(β,δ) has vertical codomain (g',ℓ')
and $(g,\ell) = (g, b\cdot\ell') = (g\cdot b, \ell') = (g',\ell')$
so (α,γ) and (β,δ) can be composed. Thus,
in $H \odot K$ we have the square at right:

$$\begin{tikzcd}
A & C & D \\
\bullet \arrow[r, "{(f,k)}"] \arrow[d] & \bullet \arrow[r, "n"] \arrow[d] & \bullet \arrow[dd] \\
\bullet \arrow[d] \arrow[r, phantom, "{(\beta,\delta)\circ(\alpha,\gamma)}"] & \bullet \arrow[d] \arrow[r, phantom, "\Downarrow\varphi"] & \\
\bullet \arrow[r, "{(h,m)}"'] & \bullet \arrow[r, "p"'] & \bullet
\end{tikzcd}$$

and therefore a square

$$\begin{tikzcd}
A & D \\
\bullet \arrow[r, "{((f,k),n)}"] \arrow[d] & \bullet \arrow[dd] \\
\bullet \arrow[d] \arrow[r, phantom, "{((\beta,\delta)\circ(\alpha,\gamma),\ \varphi)}"] & \\
\bullet \arrow[r, "{((h,m),p)}"'] & \bullet
\end{tikzcd}$$

in $(H \odot K) \odot L$.

On the other hand, in $K \odot L$ there are no nontrivial pairable squares: Nothing of the form $(\gamma,-)$ or $(\delta,-)$ or $(-,\varphi)$. Hence in $H \odot (K \odot L)$ there is no analogue of $((\beta,\delta)\circ(\alpha,\gamma),\varphi)$.