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THE MATHEMATICAL AND THEORETICAL BIOLOGY INSTITUTE - A MODEL OF MENTORSHIP THROUGH RESEARCH

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Abstract. This article details the history, logistical operations, and design philosophy of the Mathematical and Theoretical Biology Institute (MTBI), a nationally recognized research program with an 18-year history of mentoring researchers at every level from high school through university faculty, increasing the number of researchers from historically underrepresented minorities, and motivating them to pursue research careers by allowing them to work on problems of interest to them and supporting them in this endeavor. This mosaic profile highlights how MTBI provides a replicable multi-level model for research mentorship.

Acronym listing. To help the reader keep track of the various acronyms, we include a list of acronyms used in this paper with a paranthetical explanation if appropriate.

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AAAS	American Association for the Advancement of Science
AMLSS	Applied Mathematics in the Life and Social Sciences (a Ph.D. program at ASU)
AMS	American Mathematical Society
AMSSI	Applied Mathematical Sciences Summer Institute
ASU	Arizona State University
C^3	Carlos Castillo-Chavez, Director of MTBI (pronounced "C-cubed" and coined in MTBI 2000)
JBMSHP	Joaquin Bustoz Math Science Honors Program
LANL	Los Alamos National Laboratory
MTBI	Mathematical and Theoretical Biology Institute
MCMSC	Mathematial, Computational and Modeling Sciences Center
NSA	National Security Agency
NSF	National Science Foundation
PAESMEM	Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring Program
REU	Research Experience for Undergraduates
SIAM	Society for Industrial and Applied Mathematics
SMB	Society for Mathematical Biology
SUMSRI	Summer Undergraduate Mathematical Sciences Research Institute
URM	Underrepresented Minority (defined by the NSF as a U.S. citizen or permanent resident who is Latino, African-American, or Native American)

1. **Introduction.** MTBI is arguably the single most important contribution of Professor Carlos Castillo-Chavez. Those who know a little about MTBI often call it an REU (funded by the NSF and the NSA). However, anyone who has spent some time embedded in MTBI realizes that the "REU MTBI" is just one piece of a larger mentorship and revolutionary research training program. In the past decade it has grown to encompass not only graduate students but faculty and post-docs from the USA and around the world who come for training in mathematical biology, for knowledge on how to run a summer program, or both. Underneath the serious and laudable goal of training researchers is a more game-changing goal: to change the face of the scientific endeavor through MTBI participants (who are emboldened to pursue science careers through the research topics that they choose to address). MTBI has evolved into a mentorship program in the mathematical sciences that trains participants from high school to the postdoctoral and junior faculty level with great success. Between May of 2003 and May of 2011, 76 MTBI students have completed their PhDs, 49 of these from U.S. underrepresented minority groups [2, 4]. What is not apparent in these numbers is that the vast majority of undergraduate students who participate in MTBI don't meet the traditional definition of a student with "graduate-school potential." That is, they typically do not come from elite schools, do not have flawless or near-flawless GPAs, and are often from underrepresented and/or underprivileged backgrounds. MTBI has taken many of these "diamonds in the rough" and shown them that they actually *do* have what it takes to get into graduate school, to succeed in graduate school, and to have high-impact careers in the mathematical sciences. Ph.D. alumni of MTBI are in

tenured and tenure-track positions at universities like Brown University, University of Michigan, University of California - Santa Cruz, ASU, and more, as well as in permanent positions with employers such as the NSA.

Over the years, MTBI alumni have been crucial in the development and establishment of research communities of underrepresented minority (URM) graduate students at ASU (41+), Cornell University (22+), and the University of Iowa (15+) [4]. This list of universities continues to grow as graduate students now often come to MTBI to find their Ph.D. dissertation topic or to get help through critical research points in their dissertation during the summer portion of MTBI. MTBI has single-handedly doubled the number of U.S. Latinos earning a Ph.D. in mathematical biology and increased the number of those earning Ph.D.s in the mathematical sciences by 33% [4, 9]. Through the work of alumni and past faculty visitors, it has also been a catalyst for similar transformative programs such as the AMSSI,[1] co-founded by authors EC and SW at Loyola Marymount University and Cal Poly Pomona, and Miami University of Ohio's SUMSRI, founded by Dennis Davenport.[2] MTBI goes beyond simply recruiting participant URM researchers at all levels; it is revolutionizing the sciences by empowering these individuals to pursue research careers because they can set their own research agendas and work on problems that are of interest to them and their often-marginalized communities.

Building sustainable communities of mathematical scientists is not an organic process nor can one simply build a generic program in which the ingredients are simply brought together and magic happens. Rather, it is strategically and carefully planned and carried out step by step with modifications sometimes needed at many of these places. Multiple deliberate efforts intertwine, with a focus on the following points in MTBI:

1. Undergraduate students who are participating in the 8-week summer REU.
2. Returning (advanced) students or post-baccalaureates who are being helped with the transition to graduate school.
3. Graduate students who are receiving support at critical dissertation stages.
4. Graduate students—mostly from ASU's AMLSS Ph.D. program but 1–3 from other institutions—who mentor undergraduates through office hours and work on their projects and receive mentoring by the faculty.
5. Faculty members who are going through development and training in mathematical biology research and mentoring.

Developing a model for effectively addressing the challenges that limit the success of URMs across these critical transitions in the mathematical sciences and its effective implementation is at the core of MTBI. For the model to work, one must have a critical sustainable population of URMs and devoted faculty who are willing to put their lives and sleep on hold for the intense 8-week REU to help the students. Since MTBI's move to ASU, MTBI has evolved to address each of the above populations. The multiple levels of its participants thus requires mentoring at each level and there is both vertical and horizontal mentoring from within these tiers, which we refer to here as *multi-level mentoring* [2, 4, 5].

The evolution of MTBI from an undergraduate REU to a multi-level mentoring program was not the end result of many off-the-cuff decisions but was instead a strategic plan, many years in the making, that carefully formed MTBI into what

[1]The website for AMSSI is <http://www.public.asu.edu/~etcamach/AMSSI/index.html>.

[2]The website for SUMSRI is <http://www.units.muohio.edu/sumsri/>.

it has become today. MTBI is a program that is based on the training and mentoring of individuals (at all levels) through research. As will be detailed below, C^3 (and thus MTBI) has a fundamental belief in the importance of establishing a strong collaborative learning environment in which the participants can grow both academically and personally through excellent teaching and mentoring so that they can work on problems of interest to them and their communities while advancing their research [3, 14]. We will give a brief history and some details of how MTBI has evolved into such a program with such a national impact.[3] We note here that this paper is meant as an introduction to and an overview of MTBI and its many facets that together make it a unique program. Details of how one might replicate its structure, problems one may encounter in doing so, examples of the type of student that MTBI accepts, and a range of other details can be found in [2, 3, 4, 14].

2. **A needed experiment.** One of the main ideals behind the creation of MTBI was (and still is) the desire to change the landscape in the mathematical sciences. A total of 3626 Ph.D.s were awarded to U.S. citizens/permanent residents in the mathematical sciences from 1997–2002 [9]. However, only 192 (32 per year) or 5.3% went to U.S. URMs—a minuscule percentage when compared with the proportion of URMs in the U.S. population [9, 15]. The percentage of Ph.D.s awarded to U.S. URMs increased to 7.2% (363 out of 5032 or, equivalently, 45.4 per year) during 2003–2010. The presence and influence of MTBI alumni Ph.D. recipients over this period is unmatched. MTBI alumni contributed 54 of the US-URM Ph.D.s awarded just during 2005–2010 (9 per year); that is, 33% of the eight-year average increase in Ph.D.s over this window in time. MTBI alumni earned 83 Ph.D. degrees [69 U.S. citizen] between May 2005 and May 2011; that is, nearly 12 U.S. Ph.D.s per year. MTBI has contributed between 1.6% and 4.3% of the Ph.D.s in applied mathematics earned by students at U.S. institutions since 2005. C^3 stated in [4] that MTBI U.S.-URMs were awarded roughly 50% of the mathematical biology Ph.D.s each year and most of the Latino/a Ph.D.s in mathematical biology (given that 90% of MTBI URM math Ph.D.s are awarded to Latinos/as).

With the abysmal numbers of URMs earning Ph.D.s in the mathematical sciences through the early 1990s as a catalyst, C^3, with the help of Professor Herbert Medina, the urging of Professor William Velez, funding from Dr. James Schatz (NSA), and the support of Cornell University Provost Don Randal, created the 1996 "experimental program" [2, 4]. The 1996 precursor MTBI *was* the typical REU with two exceptions: (1) all of its 36 participants were URM (30 U.S. URM and 6 from Latin American countries), so it had more minorities than all of the other existing REUs combined [2], and (2) the sophomores and juniors who participated were from nonselective institutions and *not* the traditional graduate-school-bound students [4]. In addition, with women making up nearly 50% of the population in the U.S. but only approximately 30% of the math Ph.D.s [9], a conscious effort was made to keep a balance among the participants in terms of gender. These students took classes for three weeks to bring everyone to roughly the same knowledge level on various mathematical topics that were often applied to biological problems, such as discrete- and continuous-time dynamical systems, probability, stochastic processes, and a few

[3]The recognitions given to MTBI are numerous and most recently include a 2011 PAESMEM <http://paesmem.net/awards/institutions/86> and a 2007 AMS Programs that Make a Difference Award <http://www.ams.org/programs/diversity/emp-makeadiff>, <http://www.ams.org/notices/201004/rtx100400528p.pdf>, as well as a Mentor Award from AAAS <http://www.aaas.org/aboutaaas/awards/mentor/mentor2007.shtml> for C^3.

others. They also had computer labs in which they learned the basics of software such as MATLAB, Maple, and Mathematica, and had homework assignments (with graduate students and faculty holding office hours in a common room within the dorms) that kept them busy until 1 or 2 a.m. at the earliest. The students then divided into groups of six and worked for the remainder of the six-week program on reproducing the results of an existing and recent paper from the mathematical biology literature and then proposing and executing extensions of their respective models. Throughout the program, the students bonded with each other and established quite a strong collaborative learning environment, a trait that is not common to all the REUs today but has become a strong component of some of them [3, 14].

Perhaps the most impressive result of the summer program was the research project extensions proposed by the students. It wasn't that any of the resulting projects were ready for publication, but rather that a group of students that most other REUs wouldn't think of accepting into their program were actually very capable of demonstrating the necessary first steps to synthesize results from the literature and come up with their own ideas of how to improve them. This observation was key to the founding and future direction of MTBI: beginning with MTBI 1997, student groups always came up with their research topic and question, instead of having the faculty give it to them, and almost always made substantial progress on their proposed problem.

3. **The formative years.** Beginning with the the winter of 1996-97 and then every summer MTBI thereafter (1997 and on), some students from the previous year returned to participate in MTBI, attending morning advanced lectures on topics that they traditionally would not see until graduate school, such as Markov processes and bifurcation theory, and then helped in the afternoon with computer labs and at night with office hours. These 3–6 students, known as "advanced students," completed the mentoring circle in MTBI that had subsequently expanded to all levels. However, they still had their own homework to complete on a typical day. Sometimes these students would work on their own projects and sometimes they would join a "new student" group to work on a research project. This was an important step in the evolution of MTBI into its current multi-level mentoring effort. The second crucial step in the evolution of MTBI was that all students (both REU and advanced students) were required to *create* open-ended projects on which to work, in addition to conducting the traditional analysis on them and interpreting the results. The presence of students returning for a *sequential* research experience (i.e., coming to MTBI for more than one summer) involved training them as mentors together in addition to requiring students to create their own mathematical models and this sequential experience has impacted students, as many have expressed to us over the years.

Challenges abounded and centered around the following question: "How can you get undergraduate students to create and analyze an open-ended question given their limited mathematical background?" The answer is short and twofold: with (1) a collaborative learning environment and (2) a very supportive faculty and graduate student contingent that is willing to put in extensive hours of work (including all-nighters) and to let the students see how their thinking process really works when given a problem that they know *little or nothing* about. The now eight-week MTBI thus became a program of *student-driven* research because it was the students who came up with the research topics, pored over the literature for explanations of the mechanisms involved in a given problem, with the help and guidance of the faculty proposed equations that described this mechanism, analyzed the equations with techniques learned in the first few weeks of the program, and then interpreted and wrote up the results. Each student group had substantial help in developing its research idea into a concrete research question and mathematical model from the faculty as well as formulating and analyzing their research problem. It is important to remember that the typical MTBI student has not had extensive success with and exposure to this level of critical thinking, and thus it can be challenging to bring up the level of student learning to that necessary to carry out the research projects; see [2, 14] for more detailed discussion of this challenge together with a few specific examples of the students whose research trajectories began at MTBI. The student-driven nature of MTBI research projects together with the two necessary ingredients (mentioned above) are key characteristics to MTBI. An understanding of this has influenced the evolution of MTBI into its current state where faculty (both senior and junior) get retraining or reinforcement in this area and learn how to guide undergraduate research; students also benefit from this mentoring and training.

From 1997 to 2002 MTBI remained at Cornell University and continued to have an impact on 20–30 new and advanced students per year (many U.S. URMs and students from non-selective schools with minimal research opportunities). As MTBI developed a reputation as an intense “math boot camp,” professors began to send their post-baccalaureate students to MTBI in the summer before beginning graduate school. The reasons were typically to expose them to the rigors they would expect in graduate school and/or advanced topics that would be useful in training them to be ready to undertake research in graduate school in a timely manner.

4. **A new home.** In January 2003, C^3 took a 1-year appointment as an Ulam Scholar at Los Alamos National Laboratory (LANL) en route to his new position at ASU. MTBI 2003 began an adventurous 3-year stint of the program at LANL. While the structure that had been shaped over the last several years remained intact—student-driven research, advanced students, large percentages of URM and women, and heavily involved faculty and graduate student help—the logistics of running MTBI at LANL were formidable. Obtaining clearances in order to provide access to any part of LANL was difficult and much of MTBI (classes, computer labs, some office hours) took place at the local high school. The 2004–2005 programs were especially complicated, as C^3 was physically in ASU for most of the year and the dedicated staff at LANL and ASU worked tirelessly to see that the programs went as planned.

MTBI 2006 finally saw its new and current home at ASU. Over the previous few years, MTBI had further expanded its training efforts to now include graduate students and faculty. Specifically, there were now 1–3 graduate students from other institutions (such as Cornell University, Purdue University, University of Texas - El Paso, Howard University, and many others) who attended MTBI each year as advanced students but rather than working on projects with a group they focused instead on their dissertation topic or on finding a topic for their dissertation, effectively utilizing the faculty resources present during the summer MTBI. While most of the faculty who came to dedicate their time to MTBI were experts in mathematical biology or in a particular area of applied math used at MTBI, some faculty who sought *retraining* in mathematical biology had also begun coming to the program as junior faculty who needed to create strong research programs and build collaborations to fortify their tenure portfolios and move successfully through their critical pre-tenure transitions. All participants received training and mentoring in multiple ways, many of which were not seen or only began to take shape during the formative years of MTBI. For example, one of the important evolutions that had occurred was the idea of *mentor meals*. The students had always eaten dinner at the same location (whether in the cafeteria at Cornell in 1996, the fraternity/sorority house dining room at Cornell in 1997-2002, a specific local restaurant in Los Alamos in 2003-2005, or in a common area within the students' living quarters at ASU in 2006–present); however, the emphasis of these dinners began to focus on the informal mentoring that had begun to occur on a regular basis as the MTBI students, graduate students, and faculty dined together. This mentoring opportunity in a casual setting, without research notes or whiteboards, is now a recognized important component of the multi-level mentoring within MTBI.

5. **MTBI summer experience.** To fully appreciate how MTBI achieves its goal of mentorship through research, we give details of the current structure of the eight-week summer MTBI. During the first four weeks of the institute, the undergraduates and post-baccalaureates receive morning lectures from MTBI faculty and visitors on a range of mathematical and biological topics relevant to mathematical biology research—including qualitative and scientific computation of discrete and continuous dynamical systems, introductory bifurcation theory and local and global stability, linear algebra, probability and stochastic processes, pair-formation models, epidemiology, genomics and genetics. During this time, graduate-level lectures are held for the advanced students and attended by the graduate students and faculty-in-training (such topics covered have included age- and space-structured models, applications of center manifold theory to bifurcations in population biology models, etc.). The REU students receive computer training in the afternoon in Maple, MATLAB, LaTeX, and other software (such as Berkeley Madonna or XPP) related to that day's topics. Sometimes, graduate students or faculty have found it beneficial to attend the computer lab session for either a crash course or refresher course in many of these symbolic or computational software packages. Some sessions address aspects of the research process such as reading scientific literature or modeling. Additionally, the mentor meals provide a time, a place, and an informal setting for the faculty and graduate students to dine with and mentor the REU students and each other. This allows them to share graduate school and professional experiences and discuss interesting research questions and personal challenges. The shared work, shared activities, and collaborative environment enable MTBI's success.

After the mentor meals, the REU students begin work on their homework, on which they are expected to work in a collaborative environment with the support of the AMLSS graduate assistants and MTBI faculty. Students who finish early are expected to help other students with their assignments, as no student is allowed to leave until everyone has finished the assignment. Thus, students learn quickly that collaborative learning is the best way to get the work done before 3 a.m., learn new material, and reinforce previous knowledge. The advanced students, all graduate students, and the faculty participants work in the same designated area as the REU students, an arrangement that provides another mentoring opportunity from which both mentors and mentees can benefit. The rapport that develops between

REU students, AMLSS graduate students, MTBI faculty, and the other participants during these long evening sessions carries over to the research phase of the program, where it becomes a critical ingredient that allows all participants and the research groups to withstand the rigor, long hours of work, and group dynamic challenges of the research portion; see [3, 4, 14]. A few organized extracurricular activities such as trips to a baseball game, an overnight camping trip, a trip to a summer conference such as the annual meeting of SIAM or SMB, or MAA's MathFest all further solidify the bonds among the participants.

The end of the fourth week sees a transition in MTBI from coursework to research. Students self-select groups and project topics. Once the students have come up with a research idea, numerous research meetings are held during the fifth week of MTBI (with coursework portion now finished) in which all of the graduate students, faculty, and MTBI visitors are in attendance to give the student research groups feedback on their ideas and progress. This feedback is crucial for the progress of the given research project and is equally crucial for the mentoring development of the graduate students and faculty.[4] MTBI has found it essential for these groups to see the thought processes behind the advice of the experts, and for students to realize that faculty do not know all the answers. The student-driven nature of the projects allows the students to address research questions that are of interest to them and to be engaged in mathematical research. The student groups are assigned a lead faculty mentor, an assistant faculty mentor, and an AMLSS graduate mentor who work with them and guide them for the remainder of the research. Research meetings are ongoing during the sixth and seventh weeks to ensure that appropriate progress is being made, and to help students document their work through technical reports, posters, and presentations. The research groups present their research results in a university-wide colloquium (and, some years, at national and international conferences) during the eighth week.

As we have discussed, MTBI began as an REU with only undergraduate participants but over the years has expanded to include high school students, graduate students, postdocs, and faculty, all of whom work with each other to advance research. This best serves the overarching goal of changing the way the scientific research agenda is set—a vision of an agenda set by all participants in the endeavor, a group of individuals diverse not only in race, gender, ethnicity, academic major, research expertise, and interest but also in academic seniority. This requires that all levels of researchers be empowered to share ideas, make decisions, and pose research questions. While the faculty members have broad expertise, they cannot possibly cover all potential topics from either a biological or a mathematical point of view. When it comes to selecting research topics, students choose the research topics that are of interest to them—e.g., cancer, autism, ataxia, or eating disorders—because they know people or have family members who have been touched by these issues. While students come up with the topic/idea, the faculty help transform this initial idea into a feasible research question. Faculty work collaboratively and intensively, not abiding by any concept of time or normal work days to get over the steep learning curve that a topic well outside their expertise poses, with the goal of helping the students advance their research and answer some of the set of questions they set out to address from the beginning. Faculty serve to help define, identify, and refine

[4] An article by Mason Porter, a multiple-year MTBI graduate student assistant during its time at Cornell University, details some of this mentoring and his transition from mentee to mentor; see [10].

feasible research questions from these topics, but this structure helps invert what many students previously viewed as a hierarchy in terms of who sets the agenda.

In addition, MTBI has been able to draw some students from JBMSHP, the PAESMEM-winning program at ASU that trained talented high school students, many of whom were from underprivileged backgrounds. C^3 arranged for both JBMSHP and MTBI to be housed in the MCMSC to maximize the synergy between the programs. The interplay among the multiple facets of MTBI has really taken root.

6. Mentorship through research.

As MTBI evolved to its current state, so did its goals and objectives to better address the mathematical needs of the U.S., in particular as it relates to its workforce. One of MTBI's underlying goals is to expand mentorship to every level—a goal that took many years to unfold fully. C^3 also realized early on that giving students an intense summer experience is not itself enough to ensure their success years down the line. Thus, another part of MTBI's agenda was that of reinforcing the students' academic and research exposure (and success) by developing a sequential research experience (see Section 3), and students and faculty alike sometimes come for multiple summers to broaden and continually reinforce their training in mathematical biology. The REU aspect of MTBI is perhaps the most well-known, but the other components are equally crucial in MTBI's efforts to increase the number of successful Ph.D. recipients and scientists in the mathematical sciences that are U.S. URMs and that are capable and ready to impact our nation and transform the many marginalized communities from which the participants come. MTBI ensures that URMs successfully transition from undergraduate to tenure through all of these critical components, which gain leverage from each other. The other aspects of MTBI are crucial to these efforts (some are year-round efforts, whereas others take place only in the summer for 8 weeks in conjunction with MTBI's REU component). Research workshops and working groups have been held that are aimed at reinforcing MTBI collaborations between faculty, visitors, and graduate students from various years and furthering the collaborations among these individuals with more established researchers. These also serve as a venue to disseminate research with MTBI website, <http://mtbi.asu.edu/>, being used to disseminate this information along with personal e-mails. Through this component, MTBI maintains year-round research and mentoring of all MTBI participants by senior MTBI faculty and nationally and internationally renowned MTBI visitors.[5]

Over the years, MTBI's commitment to mentorship through research has transformed it beyond a summer experience to fulfill its long-term commitment to mentoring and professional development of its participants. MTBI makes it part of its responsibility to see that its alumni not only survive but thrive in graduate school and the tenure process. MTBI involves its graduates in its summer research program as advanced participants and often funds their trips to conferences. MTBI participants are connected to and network with well-known mathematicians and biologists. Most minority students are likely to be mentored by a non-minority advisor in their graduate studies, some of whom may not be aware of the challenges faced by URM students. This makes it crucial to develop a sustainable extended network for these students. MTBI provides support to students who may feel isolated in graduate school by providing summer research support (mentors and financial support) to

[5]MTBI visitors include Tom Banks, Fred Brauer, Horst Thieme, Simon Levin, and many others.

reduce the likelihood that they will quit. Similarly, it helps junior faculty successfully navigate the tenure process by helping them develop sustainable and sound research programs, become retrained (if necessary) in interdisciplinary mathematics, build collaborations, and get training in undergraduate research mentoring. MTBI frequently brings back MTBI alumni who have already entered graduate school and young faculty to help them continue their research and to serve as mentors and role models in the summer program.

7. **A sample of MTBI research.** The technical reports of the 161 research projects co-authored from 1996–2012 are achived at the MTBI website and can be found at <http://mtbi.asu.edu/research/archive>. As could be expected with such a wide range of topics, the mathematics used has drawn from nearly every aspect of the lecture portion of MTBI, including difference equations, differential equations, and stochastic processes. A key characteristic of MTBI is the *modeling* that takes place in any given project, where the students find a particular flavor of (say) a compartmental model that is reasonable, find reasonable data to estimate parameters, try different ways of doing it as dictated by what data is actually available in practice, etc. The student-driven nature of MTBI projects has produced both an impressive diversity and some definite trends in research areas addressed by the projects over the years. Many of these, with additional work by the students and faculty after the summer program, have turned into refereed publications in journals such as the present one, *Discrete and Continuous Dynamical Systems*, the *Journal of Theoretical Biology*, the *Journal of Mathematical Psychology*, and many others [2, 4]. In this section, we give a brief snapshot of some of the most common project research areas with the number of them in parentheses.

Population ecology (10): These projects have included investigations ranging from the population dynamics of the Monarch Butterfly (MTBI-08-01M, 2011) and harvesting among Yellowfin Tuna in the Eastern Tropical Pacific (BU-1511-M, 1998) to tropical montserrat flora after periodic volcanic eruptions (BU-1513-M). There is roughly an equal number of projects dealing with species in the air, the land, and the water.

Landscape ecology/pair formation (7): Projects have ranged from the modeling of forest fires (MTBI-07-02M, 2010) to understanding spatial heterogeneity's role in shaping the evolution of the European corn borer moth *Ostrinia nubilalis* (BU-1582-M, 2001). The spatial focus in these studies distinguishes them from many of the other projects, using pair-formation techniques to address the aggregate spatial heterogeneity of a landscape (the evolution of properties of adjacent patches) without requiring the thousands of state variables of a standard spatially explicit model [6].

Disease models: A large portion of MTBI projects investigate the spread of epidemics through a population. Many students have studied HIV (14) and influenza (10), and some have examined the spread of multiple strains (of these and other diseases) either within a single host or within the population (12).

Wildlife diseases (9): From the study of Chagas' disease in sylvatic cycles (MTBI-06-07M, 2009; MTBI-05-05M, 2008; MTBI-02-12M, 2005) to chytrid fungal infection dynamics in the tropical frog *Eleutherodactylus coqui* (MTBI-07-10M, 2010) and the spread of hantavirus in rodents (BU-1585-M, 2001) these projects applied epidemiological techniques to diseases in animal populations.

Population epidemiology (55): This is by far the subject area in which the greatest number of MTBI technical reports have been written. Besides HIV and the flu, other projects have investigated the spread of such diseases as tuberculosis (MTBI-07-04M, 2010; MTBI-07-06M, 2010; MTBI-04-08M, 2007; BU-1589-M, 2001), the Human Papillomavirus (MTBI-04-03M, 2007; BU-1621-M, 2002), the West Nile Virus (MTBI-01-08M, 2004), and head lice (BU-1422-M, 1997).

Within-host epidemiology (12): Some of the within-host models have involved the diseases mentioned above, but they have often focused on the effects of drug treatment on individuals. Besides HIV and tuberculosis, projects have investigated Hepatitis C (MTBI-06-03M, 2009; MTBI-05-04M, 2008) and streptococcal infections (BU-1524-M, 1999).

Other models in biology and science fields[6] **(32)**: Because the projects are student-driven, several of projects are not within the framework of the mathematical biology topics mentioned above. Some of these have included mathematical models of diabetes (MTBI-08-04M, 2011; BU-1579, 2001), a model of photoreceptor interactions (BU-1640-M, 2003), and the effects of maternal age on autism (MTBI-05-03M, 2008).

Sociodynamics (24): From the very beginning (1997), MTBI students have applied their knowledge of epidemiological modeling to social settings. In almost every year since, mathematical models have been created that examine collective behaviors within a social setting. These projects range from politics (MTBI-02-02M, 2005) to education (BU-1645-M, 2003; BU-1586-M, 2001; BU-1526-M, 2000), crime (MTBI-08-08M, 2011; MTBI-04-07M, 2007; MTBI-02-08M, 2005; BU-1508-M, 1998; BU-1504-M, 1997) cigarette use (MTBI-03-04M, 2006; BU-1525-M, 2000; BU-1505-M, 1997), and many other topics where peer pressure and influence play a role. Some of these projects dealing with social dynamics have made a surprisingly strong impact, as many are, to the best of our knowledge, among the first models or first epidemiological models of their type for the given application [5, 7, 8, 11, 12]. The multiple nonlinearities that reflect the influences of peer pressure on collective behaviors have led to complex descriptions of how these phenomena arise and persist.

8. **Discussion.**

MTBI has had a tremendous impact on not only the educational and career trajectories of the over 400 students and faculty that have participated in it over the years, but the mainstream scientific literature via the technical reports and eventual refereed publications it has produced [4]. It is a unique community in which student and faculty alike immerse themselves in an intense and rigorous collaborative learning environment and accomplish educational and research tasks and agendas that many find it hard to believe were done in a matter of 8 weeks. While many dedicated faculty, graduate and undergraduate students, and staff have contributed in small and large part to this success, it is C^3 who has been the catalyst behind MTBI and its success and continues to mentor and help advance student ideas year after year. MTBI began as a typical REU in which students are mentored to take the first steps in the direction of doing independent research that has an impact in their lives and often relates to their communities. It has evolved into a multi-level mentoring effort in which students from undergraduate (and sometimes even high school) through postdoc and faculty come to receive

[6]These disparate and broad areas are used as a single heading only because of the lack of a coherent thread of the 32 projects.

training in mathematical biology and in mentoring and are fundamentally changed by their experience. C^3 found early on that student learning and excitement are greatest when the topics come from the students' own interests, and constructing a system in which *student-driven* research, through which all levels of participant give and receive mentoring, has become a hallmark of MTBI.

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