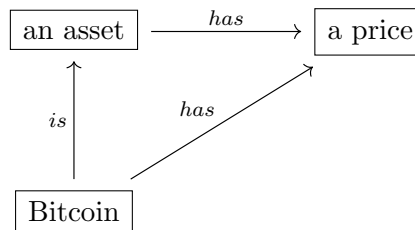


# Investigation Regarding Derivatives Trading

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Let's try to build a sound model of what "derivatives trading" would be like, especially to explore if it would be effective or not. I will follow the style of ologs in diagramming a web of concepts and their relationships to each other, which hopefully, as the outline becomes more precise, turns into an actual mathematical model or description of said thing.

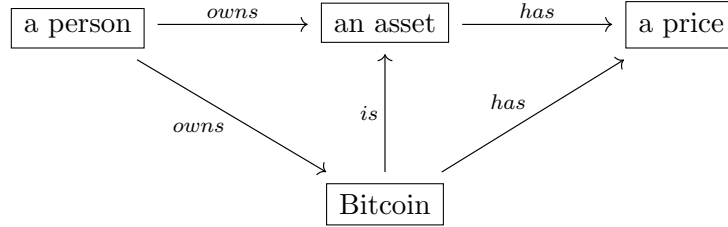


I like this as a first stepping stone in developing a view of the phenomenon. Clearly, a factor that is lacking is time. I will add time as a factor in this diagram, but I am also curious about how the original diagram could be kept, and there could be some way of embedding the temporal version of the graph inside an atemporal one of "static, general" truths. For example, I will simply add time as a variable  $t$  here, but imagine if "time" was actually another node in the diagram, and its effect on the other objects was via an arrow. For example, what if "a price at time  $t$ " was actually the product of objects "price" and "time"?

$$\boxed{\text{a price}} \times \boxed{\text{a time}} \xrightarrow{?} \boxed{\text{a price at time } t}$$

I might say this over and over repetitiously, but I am finding this a very effective way to think about building a model, as it is now clear what next "factor" ought to be brought in.

Of course, one of the essential aspects of markets we will have to include is that there are "agents" or "entities" or "people" (whatever you want to call them), who *own* some value/amount of an asset. *Trading* occurs when people make an exchange, of one asset for another, changing ownership of said assets. Let's add that in:



I feel like here is where things could get a bit more complicated, but I am choosing not to go down a ton of rabbit holes right now, regarding many tangential questions about category theory, and just focus on giving an outline of the idea. Let's say that at this point, intuitively, it seems like we can't get away with saying "an asset" anymore, but should specify that they have *an amount* of some asset,  $A$ ; also, the same goes for Bitcoin. Maybe we can vaguely think of this as "adding quantifiers" to our model, or, relocating these "concepts" to "units" (like, meters, kilograms, etc.), where their nodes now simply specify a *numerical value* or just pure data. We also should include that ownership also occurs at a time  $t$ , which opens mathematical questions I will defer about the nature of having the same variable in an object and in an arrow. Keeping with the theme of trying to "index" everything (I think) - maybe implying that these "nodes" are actually just sets, and a variable  $p$  represents a generically selected member representing that group?. Let's also include a variable  $p$  to refer to a *person*:

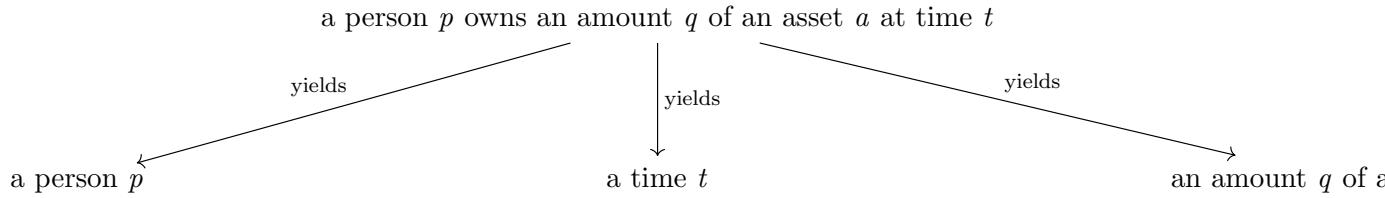
$$\text{a person } p \xrightarrow{\text{owns at time } t} \text{an amount } q \text{ of an asset } A$$

I believe in the manner of David Spivak, we allow ourselves what is basically a projection arrow, which could be labelled "yields":

$$\begin{array}{ccc} \text{a person } p & \xrightarrow{\text{owns at time } t} & \text{an amount } q \text{ of an asset } A \\ & & \downarrow \text{yields} \\ & & \text{an asset } A \end{array}$$

At this point, I might have too many ideas for where to go next, but I'll try to stay focused on the goal of representing a mathematical model of derivatives trading.

First of all, I would like to conjecture that whenever an arrow has a variable in it, we could do something analogous to currying, where that variable is seen as another input to a function. Also, my intuition is that this is a bad idea, but for now, I will explore how there could be a kind of "opposite" to a 'yields' arrow, perhaps labeled 'determines', which basically completes the "vocabulary" for us to be able to verbally describe categorical products (I think). Maybe something like this:



I was hoping the “unique arrow” in products often denoted with a dotted line could be used here and labeled “determines”, but as of now I am not sure if that actually makes sense or works.

This might be obvious, but one thing we can do now is take this out of the world of diagrams and just write it as a simple equation:

$f : P \times \mathcal{A} \times T \rightarrow \mathbb{R}$  is a function which should tell us the quantity  $q \in \mathbb{R}$  that person  $p$  owns of asset  $A$  at time  $t$ .

## 1 Review

I believe this could turn into a blog post or something if I cleaned up a first “portion” before carrying the investigation further; maybe I’ll have time to do that today. However, where I hope to go with this is:

Let’s imagine there is a function which can tell us the price of (some unit quantity of) an asset at any given time, even in the future. Of course, all we have to do is take the derivative of this function to get the tangent line of the graph / slope of the function at that point.

Derivates trading (I think) is when you automatically buy or sell a quantity of an asset in relation to whether the price is going up or down, basically.

I am interested in building a model which considers things like, how much a given trade costs in fees, and what the realistic “minimum time frame” one can monitor an asset in is, to determine if one should consider using a derivative trading strategy, and if so, what the ratio of the buy/sell amount to the rate of change of the price of the asset should be. Thanks!