Some comments on your text on Yoneda Lemma

1. The result that you obtain is not the "Yoneda Lemma" as such, but the explicit construction of the *Yoneda embedding* Y of a category C to the category of natural transformations between contravariant functors from C to Set defined as follows:

- For an object A of C, then Y(A) is the contravatiant functor C(-, A); $C \rightarrow Set$ (this functor has for codomain Set and not C as you write).
- Y associates to a morphism $m: A \rightarrow B$ the natural transformation

 $Y(m): C(-, A) \rightarrow C(-, B)$ such that Y(m)(x) = mx for each $x: X \rightarrow A$.

2. The Yoneda Lemma as such is a (not evident) consequence of this embedding. ,

Yoneda Lemma: Given a contravariant functor F from C to Set; for each object A of C the set F(A) is in bijection with the set of natural transformations from C(-, A) to F.