A Unified Comonadic Framework for Experiments, Causality, and Observers

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Abstract

We develop an extension and synthesis of several categorical approaches to physics by placing *laboratory experimentation* and *causal accessibility* under a single (co)monadic umbrella. The starting point is operational: an experimental instrument is specified by the transformations that preserve its identity as an instrument (knobs, dials, settings, readouts, and allowed reconfigurations). We model this interface as a small category C. We then interpret experiments not as morphisms $A \to B$ acting on a putative input "system," but as *structured assignments of possible observations* across configurations of C, formalized as presheaves on C. In parallel, causal accessibility is formalized using ideal completion (domain-theoretic approximation) on posets, following the causal-order viewpoint in relativistic settings. We construct a unified completion doctrine on **Pos**-enriched categories in which (i) hom-posets are causally completed by directed ideals and (ii) the instrument is experimentally completed by free cocompletion via presheaves. We explain precisely in what sense these completions are (2-)monadic, how to dualize them into (2-)comonadic form, and how the resulting coalgebras model observers whose experimental capabilities evolve with causal access. We position the framework as an extension of domain-theoretic spacetime methods [Martin and Panangaden, 2006], presheaf/topos methodologies in applied category theory [Spivak, 2014, Johnstone, 2002], and process-theoretic semantics [Abramsky and Coecke, 2004, Abramsky and Brandenburger, 2011], while addressing the ontological gap between "process-only" rhetoric and the concrete fact that experiments occur in laboratories embedded within a causal universe.

1 Introduction

A persistent foundational tension in physics concerns the status of experiments. In textbook quantum mechanics and in many categorical reformulations, an experiment is modeled as a morphism

$$f : A \to B,$$

where A is a system "entering" an apparatus and B is a system "exiting" it, sometimes with classical outcomes appended. This picture is mathematically fruitful (e.g. in categorical quantum mechanics [Abramsky and Coecke, 2004, Coecke and Kissinger, 2017]), but it is ontologically fraught: what is the object A that enters the laboratory? Where, physically, does the laboratory begin and end? How do causal horizons and observer dependence affect what it even means to specify A?

Relativistic and quantum-gravitational perspectives pull in the opposite direction: the fundamental units are events and causal relations; "systems" are at best derived, observer-dependent notions. Yet this process ontology often abstracts away the laboratory itself. The experiment is performed *on Earth*, in a local apparatus with a specific interface, within the causal structure of the universe. Any formalism that aims to be foundational must make room for this concrete fact.

This paper advances a formal view in which:

- An *instrument* is a category C determined by the structure-preserving transformations available at the interface.
- An *experiment* is a presheaf $F \in \widehat{C} = [C^{\mathrm{op}}, \mathbf{Set}]$, encoding (possibly partial) observational assignments across configurations.
- *Causal accessibility* is modeled by ideal completion (directed approximation) on posets and, crucially, on hom-posets of a **Pos**-enriched instrument.
- A *unified completion* combines causal completion and experimental completion into a single doctrine; its (co)algebras model observers embedded in, and co-evolving with, the causal universe.

Overview of Contributions

- We give an explicit categorical definition of an instrument-as-category and explain how laboratory interface constraints determine the morphisms.
- We formalize experiments as presheaves on the instrument and relate basic laboratory actions (reconfiguration, repeated trials, aggregation, coarse-graining) to standard presheaf constructions (restriction, colimits, subobjects).
- We present ideal completion as a causal approximation doctrine on posets and extend it to **Pos**-enriched categories by hom-wise ideal completion.
- We define a unified causal–experimental completion $\mathbb{G}$ on **Pos**-enriched categories and describe conditions under which it is (2-)monadic; we then explain the precise dualization by which it yields a (2-)comonad suitable for coalgebraic observer semantics.
- We prove universal properties (or provide standard proof sketches) that justify these constructions as *free* additions of (i) directed causal limits and (ii) experimental colimits.
- We give a coalgebraic semantics in which an observer is a structured map into the unified completion, interpreted physically as the rule assigning to each local configuration the space of observational possibilities consistent with the observer's causal access.

2 Background

2.1 Categories and operational interfaces

A laboratory instrument is not defined by its microscopic constituents, but by the transformations that preserve its identity as an instrument: turning a knob, pressing `RUN`, changing a filter, selecting a channel, adjusting focus, and reading a display. A catastrophic action (dropping the instrument, destroying the sensor) is excluded precisely because it exits the space of structure-preserving interface moves.

This motivates modeling the instrument interface by a category: objects are configurations, morphisms are allowed reconfigurations, and composition is sequential application.

Definition 2.1 (Instrument category)**.** An *instrument category* is a small category C whose objects are operational configurations of an apparatus and whose morphisms are interface-allowed structure-preserving transformations between configurations.

Remark 2.2 (Physics reading). The object $c \in C$ is not a physical system "in nature" but a *state of the apparatus-as-instrument*: the settings of knobs, dial positions, software modes, calibration regime, and so on. A morphism $c \to c'$ is a *procedure* (often deterministic, sometimes with controlled randomness) that transforms one configuration into another *while remaining within the instrument's operational constraints.*

2.2 Presheaves as experimental assignments

In the lab, one does not merely apply a configuration change; one records outcomes (readouts, timestamps, counts, images, traces) under various configurations. These records are (i) configuration-dependent and (ii) compatible under restriction: if a configuration c' refines c (e.g. adds extra constraints, narrows a bandwidth, fixes a gating window), then data gathered at c' restricts to data at c by forgetting the extra detail.

This is exactly presheaf behavior.

Definition 2.3 (Experiment as presheaf)**.** Given an instrument category C, an *experiment* on C is a presheaf $F : C^{\mathrm{op}} \to \mathbf{Set}$. The set $F(c)$ is the set of possible observations (or observation records) at configuration c, and for each $g : c \to c'$ the restriction map $F(g) : F(c') \to F(c)$ forgets the additional structure introduced by g.

Example 2.4 (Geiger counter schema as an instrument)**.** Consider an instrument whose interface includes a button `RUN`, a reset operation, and a discrete setting t for shielding thickness (e.g. number of plates). One may take objects to be pairs (t, mode) and morphisms to be allowed changes of t and mode; data at a configuration is (for instance) a finite list of counts. Restriction maps include forgetting fine-grained metadata (timestamps) or projecting from multi-channel acquisition to a single channel.

Remark 2.5 (Why presheaves, not just functors $C \to \mathbf{Set}$?). A covariant functor $C \to \mathbf{Set}$ would push data forward along reconfiguration, but laboratory data is typically *contravariant*: additional constraints create a *smaller* set of compatible observations, and data under stricter conditions forgets to data under looser conditions. This variance is the operational "forgetting" intrinsic to instrumentation.

2.3 Posets, causality, and ideal completion

Causality in relativistic physics is naturally encoded by partial orders: $x \leq y$ means "x can causally influence y." Domain theory adds a second order: approximation. In many operational settings, what is accessible to an observer is not the global event, but an approximating directed family of partial information. Ideal completion formalizes this.

Definition 2.6 (Directed sets and ideals)**.** Let $(X, \leq)$ be a poset. A subset $D \subseteq X$ is *directed* if it is nonempty and for all $d_1, d_2 \in D$ there exists $d \in D$ with $d_1 \leq d$ and $d_2 \leq d$. An *ideal* is a nonempty directed down-set $I \subseteq X$ (i.e. if $x \leq y$ and $y \in I$, then $x \in I$).

Definition 2.7 (Ideal completion)**.** The *ideal completion* $\mathrm{Idl}(X)$ is the poset of ideals of X ordered by inclusion.

Remark 2.8 (Physics reading). An ideal $I \in \mathrm{Idl}(X)$ is interpreted as a *consistent, directed body of partial information* about some (possibly unreachable) global situation. This matches the operational fact that observers obtain information via increasing approximations (more data, more resolution, longer integration time, additional constraints), never by accessing a God's-eye global state.

3 The Ideal Completion Doctrine on Pos

3.1 Universal property of ideal completion

Ideal completion is standardly a free construction: it freely adjoins directed suprema (dcpo structure) to a poset. We state a universal property in a form suitable for our later "causal completion" interpretation.

Definition 3.1 (dcpo)**.** A poset D is a *directed-complete partial order* (dcpo) if every directed subset has a supremum.

Theorem 3.2 (Universal property of Idl)**.** *For any poset X,* Idl(X) *is a dcpo. Moreover, the assignment $x \mapsto \downarrow x$ defines an order-embedding*

$$\eta_X : X \hookrightarrow \mathrm{Idl}(X), \qquad \eta_X(x) = \{y \in X \mid y \leq x\},$$

such that for any dcpo D and any Scott-continuous map $f : X \to D$ (i.e. preserving directed sups when they exist in X), there exists a unique Scott-continuous extension $\bar{f} : \mathrm{Idl}(X) \to D$ with $\bar{f} \circ \eta_X = f$.

Proof sketch. Standard domain-theoretic construction: Idl(X) is dcpo by directed union of ideals. Define $\bar{f}(I) = \sup f[I]$; directedness ensures well-definedness and Scott continuity. Uniqueness follows from density of principal ideals. See e.g. Gierz et al. [2003]. □

3.2 From "completion" to causal semantics

Theorem 3.2 justifies reading Idl as *freely adding causal approximation*. In a laboratory, causal accessibility constraints appear as: limited communication speed, limited bandwidth, finite integration time, and horizons (in cosmological or relativistic regimes) that prevent signal return.

In that physical reading, X captures "primitive" causal propositions/events/relations accessible in principle, while Idl(X) captures the directed, convergent refinement structure of what an embedded observer can actually accumulate.

Remark 3.3 (Monadic vs comonadic language)*.* The construction Idl is classically a *free completion* and thus naturally (2-)monadic in suitable 2-categorical contexts. Since our observer semantics is coalgebraic, we will later pass to the appropriate 2-categorical dual to present the resulting structure as a (2-)comonad. This is not a change of mathematics but a change of variance: "free completion" (monad) becomes "cofree unfolding" (comonad) under the relevant opposite/coop dualizations.

4 The Free Cocompletion Doctrine on Cat

4.1 Presheaves and free cocompletion

Presheaf categories supply colimits freely, and colimits correspond operationally to *experimental aggregation*: combining runs, pooling datasets, forming unions of contexts, and so on.

Definition 4.1 (Presheaf category)**.** For a small category C, define $\widehat{C} = [C^{\mathrm{op}}, \mathbf{Set}]$.

Definition 4.2 (Yoneda embedding)**.** The *Yoneda embedding* is the functor

$$\mathrm{y}_C : C \to \widehat{C}, \qquad \mathrm{y}_C(c) = \mathrm{Hom}_C(-, c).$$

Theorem 4.3 (Free cocompletion). *$\widehat{C}$ is cocomplete, and for any cocomplete category D, composition with y_C induces an equivalence between:*

(a) colimit-preserving functors $\widehat{C} \to D$, and

(b) arbitrary functors $C \to D$.

Equivalently, $\widehat{C}$ is the free cocompletion of C under small colimits.

Proof sketch. Standard: left Kan extension along Yoneda is colimit-preserving and yields the universal extension. See Mac Lane [1998], Johnstone [2002]. □

4.2 Operational interpretation in the laboratory

Remark 4.4 (Laboratory aggregation as colimit formation). In an actual lab, one repeats trials and aggregates results. Categorically, repeated trials correspond to coproducts (disjoint unions of runs) or, when modding out by equivalences, coequalizers/quotients. Calibration curves arise as colimits of diagrams representing measured points and their relations. The presheaf category $\widehat{C}$ contains the formal capacity to perform all such aggregations *without changing* the underlying instrument interface C.

Remark 4.5 (Variance and comonadic dualization). As with ideal completion, free cocompletion is naturally (2-)monadic. The slogan "presheaves give all possible probes" is correct, but the direction of the Yoneda map $C \to \widehat{C}$ is a *unit*, not a counit. When we later speak comonadically, we will do so in the correct 2-categorical dual setting (e.g. $\mathbf{Cat}^{\mathrm{co}}$), where the same data becomes counital.

5 The Causal–Experimental Completion

5.1 Pos-enriched instruments

To combine causal accessibility with laboratory operations, we enrich the instrument in **Pos**. Operationally, this encodes a refinement order on procedures: one procedure may be a coarse-graining (less informative) of another, or one control sequence may approximate another.

Definition 5.1 (**Pos**-enriched category). A **Pos**-*enriched category* C consists of:

- a class of objects $\mathrm{Ob}(C)$,
- for each pair x, y, a poset $C(x, y)$ of morphisms (ordered by refinement),
- monotone composition maps $C(y, z) \times C(x, y) \to C(x, z)$,
- identities $1_x \in C(x, x)$,

satisfying associativity and unit laws.

Remark 5.2 (Physics reading). The order on $C(x, y)$ can encode "more informative" versus "less informative" transformations, e.g. a high-resolution measurement procedure refines a low-resolution one; or a longer integration refines a shorter integration. This is the lab-level analog of domain-theoretic approximation.

5.2 Causal completion of an enriched instrument

Define hom-wise ideal completion.

Definition 5.3 (Causal completion of an enriched category)**.** Let C be **Pos**-enriched. Define $\mathbb{C}(C)$ to be the **Pos**-enriched category with the same objects and hom-posets

$$\mathbb{C}(C)(x, y) := \mathrm{Idl}\big(C(x, y)\big),$$

with composition induced by the universal property of Idl (extending composition from principal ideals).

Proposition 5.4. *$\mathbb{C}(C)$ is well-defined as a* **Pos***-enriched category, and the assignment $C \mapsto \mathbb{C}(C)$ is functorial on* **Pos***-enriched functors.*

Proof sketch. Composition in C is monotone, hence induces monotone maps on principal ideals. By Theorem 3.2, these extend uniquely to Scott-continuous maps on ideals; coherence gives associativity. Functoriality follows similarly. □

5.3 Experimental completion

We now add experimental colimits to the causally completed instrument.

Definition 5.5 (Experimental completion)**.** For a small **Pos**-enriched category C whose underlying 1-category is small, define

$$\mathbb{E}(C) := \widehat{C} = [C^{\mathrm{op}}, \mathbf{Set}].$$

When emphasizing the two-stage completion, we write $\mathbb{E}(\mathbb{C}(C)) = \widehat{\mathbb{C}(C)}$.

Remark 5.6 (Why **Set**-valued presheaves?). A **Set**-valued presheaf already captures the essential operational content: at each configuration, the *set* of possible records, with restriction along reconfiguration. In concrete lab practice, one often enriches further (measures, probabilities, convex structure), but the **Set**-level is the minimal substrate on which those enrichments can later be placed.

5.4 Unified completion and distributive law

We define the unified completion as the composite:

$$\mathbb{G}(C) := \mathbb{E}(\mathbb{C}(C)) = \widehat{\mathbb{C}(C)}.$$

Definition 5.7 (Unified causal–experimental completion)**.** Let C be a small **Pos**-enriched category. Define

$$\mathbb{G}(C) := \widehat{\mathbb{C}(C)} = [\,\mathbb{C}(C)^{\mathrm{op}}, \mathbf{Set}\,].$$

To treat $\mathbb{G}$ as a single (2-)monad/comonad rather than just a composite construction, one typically asks for a distributive law between the causal and experimental doctrines. We record a standard form suitable for our purposes.

Definition 5.8 (Distributive law, schematic)**.** A *distributive law* of $\mathbb{E}$ over $\mathbb{C}$ is a natural transformation

$$\lambda : \mathbb{C} \circ \mathbb{E} \Rightarrow \mathbb{E} \circ \mathbb{C}$$

satisfying compatibility axioms with the (2-)monad structures of $\mathbb{E}$ and $\mathbb{C}$.

Remark 5.9 (Interpretation of λ). Operationally, λ says that "first forming experiments, then causally completing" is coherently comparable to "first causally completing, then forming experiments." In the lab, this is the statement that (i) causal refinement of procedures and (ii) experimental aggregation of datasets can be interleaved without ambiguity, provided the refinement order and the experimental restriction maps are chosen compatibly.

Proposition 5.10 (Existence in common cases)**.** *In many concrete instrument models (e.g. when hom-posets are algebraic and experimental restriction maps preserve directed joins appropriately), a distributive law λ exists, making $\mathbb{G}$ a well-behaved composite (2-)monad on the appropriate 2-category of* **Pos***-enriched categories.*

Proof sketch. Standard: one constructs λ using the universal properties of Idl and $\widehat{(-)}$ and checks coherence via density/Yoneda. The needed hypotheses ensure that the interaction between directed joins (causal refinement) and colimit formation (experimental aggregation) is stable. □

5.5 From monads to comonads (the variance correction)

The two free constructions $\mathbb{C}$ and $\mathbb{E}$ are naturally monadic (free completion doctrines). Observer semantics, however, is typically coalgebraic: an observer carries a state of information that *unfolds* under interaction. We therefore make the variance precise.

Remark 5.11 (2-categorical dualization). Let $\mathcal{K}$ be a 2-category in which $\mathbb{C}$ and $\mathbb{E}$ are 2-monads. Then on the 2-category $\mathcal{K}^{\mathrm{co}}$ (same objects and 1-cells, 2-cells reversed), a 2-monad on $\mathcal{K}$ corresponds to a 2-comonad on $\mathcal{K}^{\mathrm{co}}$. Thus, without changing the underlying construction, we may regard $\mathbb{G}$ as a 2-comonad on a suitable dual 2-category, and speak of its coalgebras.

This is the precise mathematical form of a conceptual shift: *completion as free addition of structure* (monadic) becomes *unfolding of accessible structure* (comonadic) when interpreting observers.

6 Observers as Coalgebras

6.1 Coalgebras and observational semantics

We now define observers as coalgebras for the (dualized) unified completion.

Definition 6.1 (Observer coalgebra)**.** Fix a 2-category $\mathcal{K}$ of **Pos**-enriched instrument categories and suitable functors. Let $\mathbb{G}$ be the unified completion, regarded as a 2-comonad on $\mathcal{K}^{\mathrm{co}}$. An *observer* is a $\mathbb{G}$-coalgebra (C, γ), i.e. an instrument category C equipped with a structure map

$$\gamma : C \to \mathbb{G}(C)$$

satisfying the coalgebra axioms (counit and coassociativity) relative to the 2-comonad $\mathbb{G}$.

Remark 6.2 (Physics reading of γ). The map γ is the rule by which an observer, situated in a local instrument, assigns to each configuration the space of *available* observational possibilities, taking into account both: (i) the causal refinement structure on procedures (what can be stably approximated), and (ii) the experimental aggregation structure (what can be consistently recorded and combined). The coalgebra laws enforce that the observer's assignment is stable under "unfolding": repeated refinement of causal access and experimental capability is coherent.

6.2 A basic structural consequence

Coalgebra laws typically imply a "consistency under restriction and refinement" property. We state one form that is close to laboratory practice.

Proposition 6.3 (Stability under refinement)**.** *Let (C, γ) be a $\mathbb{G}$-coalgebra (observer). Then for any reconfiguration $g : c \to c'$ in C and any causally refined procedure represented in $\mathbb{C}(C)(c, c')$, the observational assignments selected by γ commute with restriction along g and with directed refinement in the hom-posets, in the sense that observational data is compatible with both operational forgetting (presheaf restriction) and causal approximation (ideal inclusion).*

Proof sketch. Unpack the coalgebra axioms in the presence of the distributive law of Proposition 5.10. Coassociativity expresses compatibility with iterated completion; counitality expresses that observed data agrees with realized configurations. These translate to commutation of restriction maps with directed joins and inclusions. □

7 Comparison to Existing Frameworks

7.1 Martin–Panangaden causal domains

The causal-domain program models spacetime using order-theoretic and domain-theoretic structures, emphasizing approximation and causal accessibility [Martin and Panangaden, 2006]. Our causal completion $\mathbb{C}$ extends this idea from a global causal poset to the *hom-posets of an instrument*: the observer's controllable procedures are themselves subject to approximation and horizon constraints (finite runtime, finite bandwidth, finite precision).

7.2 Spivak and presheaves as structured data

Spivak's applied category theory uses presheaves to model structured data on schemas and contexts of information [Spivak, 2014]. We retain the presheaf mathematics but shift the semantics: the base category is not a database schema but an *instrument interface*. Thus $\widehat{C}$ is not "all datasets on a schema" but "all experimental assignments compatible with the instrument." Our additional causal completion layer has no analogue in Spivak's work.

7.3 Abramsky–Coecke process theories and contextuality

Categorical quantum mechanics models physical processes as morphisms in dagger symmetric monoidal categories [Abramsky and Coecke, 2004]. The sheaf-theoretic contextuality program uses presheaves over measurement contexts to encode (non)existence of global sections [Abramsky and Brandenburger, 2011]. Our framework borrows two insights: (i) experiments are compositional and diagrammatic, and (ii) presheaves are natural carriers of contextual observational structure. However, we depart at the ontological point you have emphasized: rather than presuming a system-object A entering the lab, we take the lab's operational category C as primitive and represent "systems" only through the observer's causally constrained experimental assignments.

8 Conclusion and Outlook

We presented a unified causal–experimental completion $\mathbb{G}(C) = \widehat{\mathbb{C}(C)}$ for **Pos**-enriched instrument categories and explained its observer semantics via coalgebras of a dualized (2-)comonad. The central conceptual shift is that an experiment is not an arrow acting on an input system but a structured presheaf assignment over the configurations of a laboratory instrument, constrained and unfolded by causal accessibility.

Several directions are immediate:

- enrichment of presheaves to probabilistic/convex settings (Markov categories, effectus theory),
- connections to AQFT (nets as functors from causally-indexed posets into algebras),
- explicit case studies (interferometry, Stern–Gerlach, cosmological observation) formulated as coalgebras,
- higher-categorical refinements (2-comonads, profunctorial semantics, internal sheaves).

A Technical Appendix: Coherence, Enrichment, and Dualization

A.1 Enriched Yoneda and restriction

For **Pos**-enriched categories, one often works in a 2-category $\mathbf{Cat}_{\mathbf{Pos}}$ whose 2-cells are order-enriched natural transformations. The presheaf category $\widehat{C}$ is formed from the underlying 1-category, but the enrichment can be reintroduced by ordering natural transformations pointwise when appropriate. This is sufficient for many "laboratory" interpretations, where refinement is expressed at the level of procedures (hom-posets) and compatibility is expressed at the level of data restriction.

A.2 A schematic counit/comultiplication

Because $\mathbb{E}$ and $\mathbb{C}$ are free completions, their canonical maps are units (e.g. Yoneda $\mathrm{y}_C : C \to \widehat{C}$ and principal-ideal embedding $\eta_X : X \to \mathrm{Idl}(X)$). In the dual 2-category (reversing 2-cells), these become counits. The comultiplication arises by iterating completion and using the universal properties to define canonical comparison maps:

$$\widehat{C} \to \widehat{\widehat{C}}, \qquad \mathrm{Idl}(X) \to \mathrm{Idl}(\mathrm{Idl}(X)),$$

again understood in the appropriate 2-categorical variance. This is the correct mathematical expression of "unfolding observational capability."

A.3 A guiding commutative diagram

The unified completion is organized by the following schematic diagram (where arrows indicate canonical unit maps of the monadic viewpoint):

$$\begin{array}{ccc} C & \xrightarrow{\;\eta_C\;} & \mathbb{C}(C) \\ & \searrow^{\mathrm{y}_C} & \downarrow{\scriptstyle \mathrm{y}_{\mathbb{C}(C)}} \\ & & \widehat{\mathbb{C}(C)} = \mathbb{G}(C) \end{array}$$

Operationally:

- η_C inserts each primitive procedure as a principal causal ideal (causal approximation),
- $\mathrm{y}_{\mathbb{C}(C)}$ inserts each causally completed configuration as a representable experiment (ideal probe),
- $\mathbb{G}(C)$ then contains all colimit-generated experimental assignments consistent with causal refinement.

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