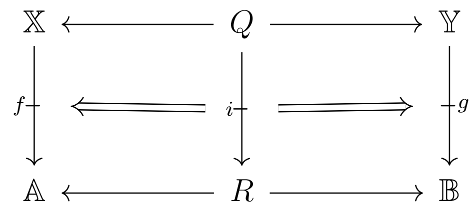
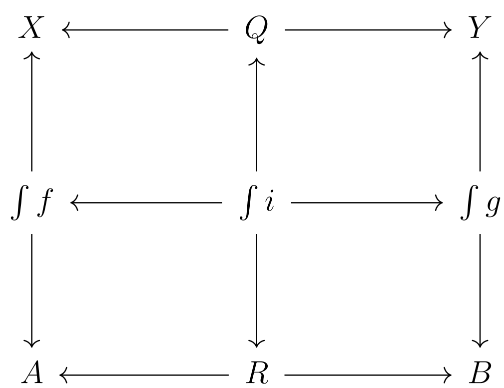
**Theorem 1.** Let $Q : \mathbb{X} \,\|\, \mathbb{Y}$ and $R : \mathbb{A} \,\|\, \mathbb{B}$ be two-sided fibrations, and let $f : \mathbb{X} \,|\, \mathbb{A}$ and $g : \mathbb{Y} \,|\, \mathbb{B}$ be profunctors. Let $i : Q \,|\, R$ be a profunctor with transformations to $f$ and $g$, i.e. a span of profunctors from $Q$ to $R$.



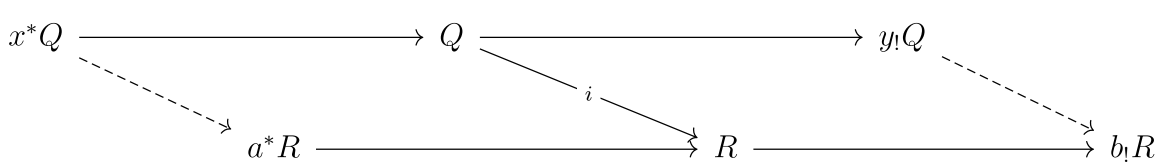

Then the induced span of discrete two-sided fibrations $\int i$ is a two-sided fibration.

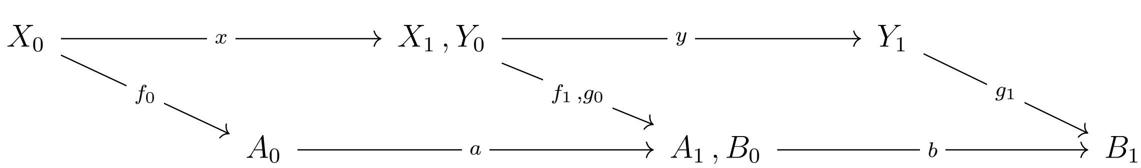



*Proof.* Let $i : i(Q, R)$ lie over $f_1 : f(\mathrm{X}_1, \mathrm{A}_1)$ and $g_0 : g(\mathrm{Y}_0, \mathrm{B}_0)$. Let $(\mathrm{x}, \mathrm{a}) : f_0 \to f_1$ be a morphism in $\int f$, i.e. a commutative square; and let $(\mathrm{y}, \mathrm{b}) : g_0 \to g_1$ be a morphism in $\int g$.

Action by cartesian morphisms is functorial: there is a canonical morphism $(\mathrm{x}, \mathrm{a})^* i : \mathrm{x}^* Q \to \mathrm{a}^* R$, forming a commutative square $(\mathrm{x}, \mathrm{a})^* i \to i$ in $\int i$ over $(\mathrm{x}, \mathrm{a})$ in $\int f$.

Dually, action by opcartesian morphisms is functorial: there is a canonical morphism $(\mathrm{y}, \mathrm{b})_! i : \mathrm{y}_! Q \to \mathrm{b}_! R$, forming a commutative square $i \to (\mathrm{y}, \mathrm{b})_! i$ in $\int i$ over $(\mathrm{y}, \mathrm{b})$ in $\int g$.




This defines a left action of $\int f$ and right action of $\int g$ on $\int i$. Hence $\int i$ is a discrete two-sided fibration from $\int f$ to $\int g$. $\square$