Emergent Feedback Loops from Crosstalk in Directed Labeled Graphs

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We study *emergent feedback loops*: loops that appear only when two signed directed graphs are glued along shared vertices, with neither graph containing them individually (see Figure 1). Our motivation comes from crosstalk between cellular signaling pathways, which rarely operate in isolation. When distinct modules share components or cross-regulate, new feedback loops can emerge in the combined network, with significant consequences for sustained signaling, growth control, developmental patterning, and disease progression. We study feedback loops using a generalization of graph homology with coefficients in a commutative monoid, and describe the emergence of new loops under composition of 'open graphs' via a variant of the Mayer–Vietoris exact sequence in this setting. We quantify the *degree of emergence of a feedback loop* via a monoid-grading on the free category generated by graphs This presentation is based on [1].

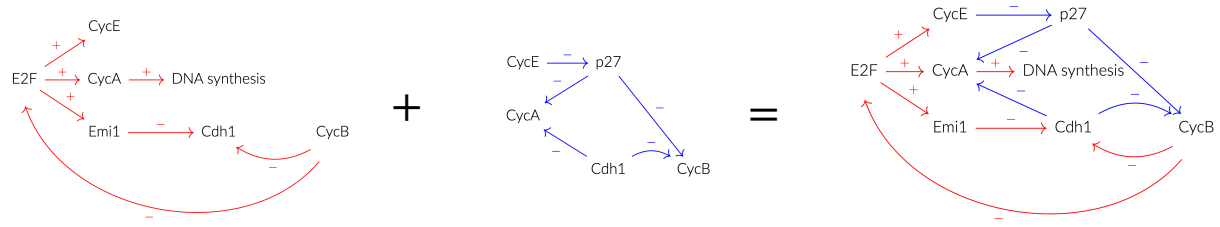

Figure 1: Below are two regulatory modules essential for DNA synthesis. Neither contains a feedback loop on its own, but composing them along their shared nodes—the proteins CycE, CycA, Cdh1, and CycB—produces loops that were absent before. Every such loop mixes both colors (red and blue); no monochromatic directed cycle exists. We call these emergent feedback loops.

[1] Motifs and Emergent Feedback in Labeled Graphs (John C. Baez and Adittya Chaudhuri), 2026, arXiv:2506.23375.