TOWARD HYBRID MODELING

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1. Overview

Many of the most urgent social, economic and public health problems facing humanity require dynamic simulation modeling for a well-informed response. Sadly, most current models are monolithic one-offs: labour intensive to build, difficult to reuse or adapt, and reliant on proprietary software that researchers are not free to improve. Our team has been developing a flexible open-source modeling framework that promotes transparency, reproducibility and scientific rigor by allowing model components to be shared and changed [1, 4, 5, 20]. This software is based on sophisticated mathematics from applied category theory, but it comes with a graphical web-based interface that requires no mathematical expertise to use.

In the current project we plan to develop software that combines two widely used simulation techniques: "agent-based models" and "stock and flow models". We call this blend of techniques "hybrid modeling".

An **agent-based model** or **ABM** describes the dynamics of one or more open populations of individual agents interacting with each other and with an environment in which they are embedded [13]. A **stock and flow model** uses ordinary differential equations to describe the change of continuous quantities in time as they flow from one "stock" to another [21]. By a **hybrid model**, we mean an ABM that combines discrete updating of agent state with continuous dynamics described by one or more stock and flow models.

In our 2024 Mathematics for Humanity research-in-groups project we successfully developed a category-theoretic framework for ABMs based on team member Brown's work [6]. We then used this framework to create a powerful software system for developing ABMs [1]. The system is now being used for research in epidemiology, and as a teaching tool. However, this system does not yet support hybrid modeling.

In 2022, several of our team members joined other collaborators to create a software system for stock and flow models [4, 20]. In subsequent years they created a web-based interface for this system [5, 15], which is now being used for research and teaching—teaching not only at the university level, but also in high school math clubs focused around minority youth in Toronto. However, this software does not allow the creation of ABMs.

In the proposed project, we plan to develop the mathematics and then the software required for a disciplined approach to hybrid models. One key feature will be the ability to "turn on" continuous dynamics of various kinds in various specific states, and disable them in other states. To give an example of great significance in gestational and Type 2 diabetes: when a woman becomes pregnant, but only then, fetal growth and her body weight evolve according to specific dynamics studied in recent research by Thomas and others [22].

We then plan to implement models that demonstrate the capacity of our software in two spheres: public health (gestational and Type 2 diabetes) and economics (labour shocks as the economy decarbonizes).

2. Objectives

The main objectives of our proposed work are to:

(1) Support hybrid modeling by developing a mathematical framework, based on category theory, that combines
- **ABMs** based on stochastic double pushout rewriting [6],
- **stock and flow models** as formalized using the mathematics of structured and decorated cospans [4, 5].

(2) Use this new mathematical framework to create software that unifies
- the existing AlgebraicABMs software for ABMs [1],
- the existing StockFlow software for stock and flow models [20].

(3) Implement models to test and refine our theoretical and software framework in two spheres:
- **Public health:** a model of the "vicious cycle" of gestational diabetes and the consequent risk of Type 2 diabetes in the mother and offspring, based on previous research by team member Osgood [9, 10, 16].
- **Economics:** a model of how the transition to a low-carbon economy can cause shocks in the labour market, based on expertise of team member Haaga and research at the Institute for New Economic Thinking [7, 19].

In more detail, we plan to:

(1) Develop the theory needed to integrate stock and flow models with rewrite-based ABM dynamics. A key requirement is to support: i) enabling or disabling operation of a stock and flow model based on an agent's state, ii) triggering transitions between discrete agent states in a manner depending on the value of some stock in a stock and flow model, iii) changing stocks discontinously when an agent transitions between discrete states.

(2) Extend and integrate two software packages this team has already built, namely AlgebraicABMs [1] and StockFlow [20]. Both packages are part of the same AlgebraicJulia software ecosystem [2], and thus built upon a common abstraction, "attributed C-sets" [18], which allows for database modeling from a category-theoretic perspective. Their extension and integration will be guided by the theory in step (1). The translation of this theory into software is promising because AlgebraicJulia has been designed to compute with categorical abstractions [3] that will play a central role in the theory of hybrid models. The construction of hybrid models will be well-documented, with illustrative tutorials to inspire practicing modelers to try applying this framework to their domains.

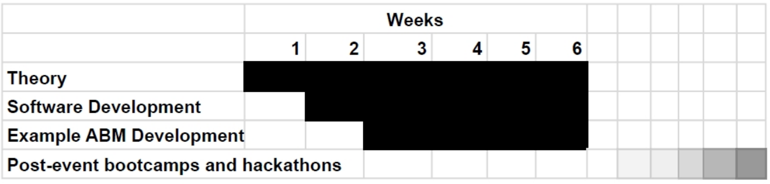
Moreover, the team has developed two graphical interfaces [8, 15] which already support the real-time, web-based collaborative design and use of stock and flow models. We can extend these to provide a graphical interface for hybrid models.

(3) Implement two or more hybrid models to test our new software. In the sphere of public health, we plan to implement a hybrid model of gestational and Type 2 diabetes based on work of team member Osgood [9, 10, 16]. Here the hybrid ABM will incorporate both discrete dynamics associated with state charts (a commonly used formalism in ABMs) and stock flow models associated both with individual-level quantities, such as body weight, fetal growth rate, and insulin levels, and aggregate quantities such as lifeyears, and quality adjusted lifeyears.

In the sphere of economics, we will construct a hybrid model of shocks to the labour market caused by decarbonization of the economy, based on team member Haaga's expertise and work done at the Institute of New Economic Thinking (INET), where he is a postgraduate. The labour market would be modeled using a variant of INET's Labour ABM [19]. Labour demand in the model would be driven by a stock-flow model of funds flowing through from final demand through intermediate goods, as specified by an Input-Output matrix [7]. Final demand would be specified using an assumption on the labour and capital shares of income together with empirically estimated Engel curves of the changing shares of expenditure by sector as a function of total household income.

3. The Plan of Work

We hope to meet from May 15 to June 26, 2025. Baez, Haaga, Li, Osgood and Waites will attend for the whole duration, Brown for two weeks during May 15 to June 1, Patterson for three weeks from June 7th to June 26, and Libkind for one week.



Event timeline, showing successive staging of work

4. The Team

The team consists of eight members, but Baez and Waites live in Edinburgh, so we need travel and lodging for only the six in boldface:

(1) John Baez: mathematical physicist and applied category theorist at the University of Edinburgh; helped develop the "structured cospan" framework widely used in AlgebraicJulia [3], and helped apply this to develop software for stock and flow models [4, 5].
(2) **Kristopher Brown**: research software engineer at the Topos Institute; led software development for ABMs in AlgebraicJulia [6, 25].
(3) **Owen Haaga**: graduate student at the Institute for New Economic Thinking in Oxford; studies new modeling techniques for labour market dynamics with applications to the post-carbon energy transition; worked for the Joint Economic Committee of the United States Congress.
(4) **Xiaoyan Li**: doctoral student in Computer Science at the University of Saskatchewan with extensive peer-reviewed contributions in health applications of dynamic modeling; lead developer of the StockFlow package using AlgebraicJulia for stock and flow health models [4, 5, 14, 20].
(5) **Sophie Libkind**: applied category theorist at the Topos Institute; developed general framework for composing dynamical systems [11] and applied it to epidemiology [12]; principal investigator for funded ARIA proposal on "Double Categorical Systems Theory for Safeguarded AI".
(6) **Nathaniel D. Osgood**: computer scientist and public health modeler at the University of Saskatchewan with dozens of peer-reviewed publications on ABMs; provincial director of COVID-19 modeling in Canada; expert on models of gestational and Type 2 diabetes [9, 10, 16]. Taught dozens of courses worldwide on ABMs for public health [17]. Contributed to development of StockFlow [4, 5, 14, 20].
(7) **Evan Patterson**: applied category theorist at the Topos Institute specializing in scientific computing, software systems and data science; helped design the AlgebraicJulia framework [2] and create software using this framework for both stock and flow [4] and Petri net [12] models of epidemiology.

(8) William Waites: computer scientist and member of the Digital Health and Biomedical Engineering research group at the University of Southampton; developed compositional and rule-based models of disease transmission [23, 24].

5. Budget

Total budget including travel, lodging and per diem: £28,841. If travel expenses from outside the UK are capped at £800 per person, this is reduced to £27,760. Details are elsewhere in our submission material.

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