CLIMATE, EPIDEMICS, QUANTUM, LANGUAGE, ROBOTICS…

WHAT IF CATEGORY THEORY WERE A THEORY OF EVERYTHING

BY CLÉMENTINE LAURENS

Many mathematicians speak of it in reserved, even critical, terms. *"When category theory was first developed, some went so far as to say that such a level of abstraction was absurd,"* recalls David Spivak, of the Topos Institute, in the United States. It is therefore all the more striking to see it mobilised today for very concrete problems in physics — but also in robotics, linguistics, epidemiology. "Applied category theory" is becoming a flourishing oxymoron: a scientific journal is now devoted to it, and the 9th annual conference on the subject will take place this July in Estonia. *"In mathematics, even when one actively tries to do things utterly disconnected from the real world, someone always ends up finding applications,"* smiles the computer scientist Jade Master, who uses these objects to model the spread of diseases.

Categories? John Baez, a researcher at the University of Edinburgh, admits: *"In this field of mathematics, the definitions can at first seem mysterious."* The principle itself is relatively simple: a category is just a set of objects, linked to one another by arrows. The nature of the objects doesn't matter — category theory focuses on the dynamics described by the arrows. And it is precisely this that allows it to connect categories that appear very different, provided they share the same dynamics.

REPRESENTING REALITY

The theory was originally developed in the 1940s to allow rigorous transfers of ideas between, in particular, topology — the science of shapes — and algebra — the science of structures. *"Spotting similarities across different areas of mathematics is natural. For centuries we have been using analogies and transposing ideas. But before the 1940s, no one had ever imagined that these ways of moving from one domain to another could themselves constitute mathematical objects!"* emphasises the mathematician Valeria de Paiva, of the Topos Institute.

The strength of the theory is that it allows its arrows to be composed in a coherent way, combining them to describe processes that unfold sequentially or in parallel. So it is not so surprising that it was quickly taken up in work on the foundations of mathematics — an activity that consists of unfolding arguments — and in computer science: this is how, for example, the Haskell programming language was born, still in use today. Linguistics, too, has proved fertile ground for category theorists. But the applications go well beyond that. *"In science, a great many notions and systems can be written as a series of instructions to be carried out one after the other or simultaneously. A cooking recipe is exactly that! Categories are therefore a natural representation of reality,"* argues the physicist Bob Coecke, of the University of Oxford, who has proposed a refounding of all of quantum mechanics on this formalism — which subsequently became a key language of quantum-computing technologies. *"Categories are an excellent substitute for logic when it comes to structuring scientific reasoning,"* David Spivak agrees. *"Logic is slow, it is laborious. When I discovered category theory, I was amazed by its conciseness: you can represent such a quantity of information in such little space! It's 'just a language', yes, but it is precisely because it is a pertinent and extremely concise language that it is an invaluable support for thought."*

Recently, the applications have taken on another, more practical dimension. The proof: →

IN LINGUISTICS

"That's grammar!" exclaimed the mathematician Joachim Lambek when he came across the application of category theory to quantum teleportation. He had previously — and independently — used categories in linguistics to relate the structure of a sentence to its grammatical correctness. *"Now, the category he was using for that was exactly the same as ours in quantum mechanics!"* recounts Bob Coecke, who then went on, with others, to develop "DisCoCat" — a mathematical framework linking the structure and meaning of language. *"It gives a very natural representation that combines grammar and meaning,"* the theorist assures.

AN ABSTRACT LANGUAGE THAT IS STARTING TO FIND CONCRETE APPLICATIONS

IN QUANTUM MECHANICS

In the early 2000s, the physicist Bob Coecke and the computer scientist Samson Abramsky developed a refounding of quantum mechanics based on category theory. *"In this very abstract language, we manage to bring out first principles, and to reconstruct properties such as quantum teleportation,"* Bob Coecke explains. *"Things that were extremely difficult in the standard formalism become easy thanks to categories."* This rewriting made it possible to develop a particularly intuitive graphical language, the ZX-Calculus, which still plays a key role in the development of quantum computing.

the theory is even starting to make inroads into epidemiology. "When Covid struck Canada, I was working for the public health authorities on modelling and managing the epidemic," recalls Nathaniel Osgood, of the University of Saskatchewan, in Canada. "And I painfully observed how underperforming our tools were compared with what the category-theoretic formalism could have offered. Categories are so well suited to these situations where you have to constantly update models, aggregate data, include new parameters…" Others are working on projects in biology, robotics, materials science — and even music: Bob Coecke is developing a tool to "generate 'categorical music' with a quantum computer". Quite a programme! David Spivak rejoices: "I have always thought that mathematics should be able to help us explain reality."

And in climatology? Category theory seems an obvious choice for representing the workings of a system as complex as the climate. "Some category theorists would love to have a go at it, but for my part I haven't yet met any climatologists eager to dive in," notes John Baez. "To my knowledge, this theory is not used by my colleagues," confirms Freddy Bouchet, director of the Laboratory of Dynamic Meteorology, in Paris.

A BIT FORBIDDING

For there is an obstacle — a classic difficulty in interdisciplinary fields, here singularly heightened: the entry cost into this new formalism is high, and it puts off many scientists. "Category theory is sophisticated mathematics that requires time and effort to learn," acknowledges Valeria de Paiva. At the start, progress is slow, which — in the absence of any guarantee of results — can be a deterrent. "I indirectly discovered categories in the 1980s, during a course on the foundations of programming languages, and I was immediately fascinated," recalls Nathaniel Osgood. "I had a deep intuition that it could be useful to me, but I wasn't sure the effort needed to really get into it was worth it. It took me more than thirty years to finally dig into

IN ROBOTICS

Designing a robot requires aggregating a great many decisions on the choice of algorithm, the physical design of the robot, the consideration of ethical or legal constraints… *"When I started in the field, I thought no language allowed all of that to be taken into account in a unified way,"* recounts Gioele Zardini, a roboticist at MIT. *"And then I discovered categories. For each problem, you use representations in the form of boxes, and the categorical formalism allows them to be combined. It's very simple, but very powerful — this universal language!"*

IN EPIDEMIOLOGY

How does a disease spread depending on the population affected, hospital capacity, the potential arrival of vaccines or variants? *"Traditionally, this is modelled with classical computer code, but it can be difficult to change something after the fact, and even harder to combine two models representing different aspects of the problem with each other,"* explains the mathematician John Baez. Jade Master, of the Coherence Research Limited laboratory, also testifies to the power of this language: *"Using categories, I am modelling the spread of an epidemic across the Glasgow metro network. And I observe how it propagates depending on which station closures are decided."*

it — and discover all the potential this theory has for epidemiology."

Hence the importance of showcasing what categories have already achieved, urges the roboticist Gioele Zardini, at MIT: "Simply saying 'Look, this category describes your problem remarkably well' will never convince an engineer. To get a new tool adopted, you have to show that this new description is a key to solving the problem." Bob Coecke insists: "For us to talk about 'applied category theory', I want there to be a potential for industrial applications — and that is the case in quantum computing!" Jade Master, for her part, tempers the enthusiasm: "Right now there is quite a bit of sensationalism around applied category theory, but it should be remembered that this is not a magic formula either. It helps to organise information, reveal structures, clarify ideas… It's powerful, but it won't answer every question."

That said, one of its most fascinating applications may not be practical — but philosophical. For a new language always offers a new perspective. *"Categories invite us to think of the world in terms of what 'happens' rather than what 'is',"* sums up Bob Coecke. Enough to revive an old debate going back to Antiquity, between Parmenides on one side — defender of a static reality and a reductionist vision in which things can be understood only by uncovering their elementary constituents — and Heraclitus on the other, for whom the world is nothing but process, where nothing remains, everything moves, everything is eternally in becoming. *"Category theory teaches us to forget reductionism and fully embrace relationism, as Leibniz advocated,"* analyses Bob Coecke. After centuries of science dominated by the vision of Parmenides, might it be time for Heraclitus's revenge?

OUR SOURCES

John Baez et al., *arXiv* (2023); John Baez et al., *AMS* (2022); Tai-Danae Bradley, *arXiv* (2018); Brendan Fong et al., *arXiv* (2018). Find all our sources at epsiloon.com/sources. All quotations are taken from interviews conducted by *Epsiloon*.