Open Challenge

1. **[Conjecture:** Consider that rhyme can be thought of as "sonic homomorphism".**]**

1.1. By "sonic homomorphism", I mean that there is *some* correspondence or *similarity* between two words, which is usually *not* absolute equivalence.

1.1.1. For example, we could either say that "milk" *rhymes* with "milk" as a sort of *identity rhyme*, or we could exclude this scenario.

1.1.2. Other than that, to say that "milk" rhymes with "ilk" appears to mean that there is a subpart of the two words which is "equivalent". In this case, it is "ilk".

1.1.3. But this opens the question, is there a reason to define "subparts" as substrings, or, what if they are syllables?

1.1.3.1. Let's assume for now that rhyme actually occurs on the syllabic level.

1.1.3.2. That means that rhyme is *co-extensive* with collections of syllables (if my use of this term is correct. I mean that as a "base case", we define rhyme as a Boolean function between two elements of the type or set Syllables. And then, we define that a *tuple* of syllables rhymes with another one in terms of a rhyme condition on each of the syllables.)

1.1.3.2.1. One interesting question this opens is if rhyme should be defined as the final syllables rhyming, or if we can have syllables in the middle or beginning of words which we consider a kind of rhyme, like "batman" and "catdog", where 'bat' rhymes with 'cat'.

1.1.4. And 1.1.2 opens the question, how does changing or loosening our definition of "equivalence" allow greater variation of "rhyme"?

1.1.4.1. For example, in rap music, there is often use of something called a "slant rhyme".

1.1.4.2. This could include as an example "wannabe" with "wildebeest".

1.1.4.3. These words do not *strictly* rhyme, but in the context of musical performance, have a similar sound which has a comparable effect to *'strict rhyme'*.

1.2. Let's say that so far, our model of rhyme is the following. This is a preliminary model which is meant to be updated through trial and error and an iterative design process.

1.2.1. First, we can define *rhyme* as a condition on syllables. Syllables break apart into tuples of phonemes. Until we broaden our definition, let's keep a simple definition that two syllables rhyme if everything after the initial consonant cluster is identical.

1.2.2. So, the word "margarine" is this tuple: ('M', 'A', 'R', 'G', 'A', 'R', 'I', 'N', 'E'), where each letter l is in $\mathbb{L}$, the letters of the Roman alphabet used in English.

1.2.3. Let's call the "head" the *initial consonant cluster*, and the "tail" everything that follows it. For the syllable "mar", the head is "m", and the tail is "ar". According to our definition, any single-syllable string ending in "ar" counts as rhyming with "mar".
[1]

So maybe we can write this out a bit better now:

1.2.3.1. We have the alphabet of English, which I'll denote $\mathbb{L}_E$, which expresses "the version of the Roman alphabet specifically used for English" (where $\mathbb{L}$ stands for 'Latin'). This well-known set contains the 26 letters of the English alphabet:

$$\mathbb{L}_E = \{\text{A}, \text{B}, \text{C}, \text{D}, \text{E}, \text{F}, \text{G}, \text{H}, \text{I}, \text{J}, \text{K}, \text{L}, \text{M},$$

[1] Chinese is a language where the syllables are often analyzed this way:

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In Chinese linguistics, there are specific terms for the initial consonant and
the remainder of the syllable:

• Initial (声母 shēngmǔ): This refers to the initial consonant of a syllable.
• Final (韵 母 yùnmǔ): This refers to the remainder of the syllable after the
  initial consonant. The final can be further divided into:
    – Medial (介  音 jièyīn): The transitional sound between the initial and
      the main vowel (if present)
    – Nucleus (主要元音 zhǔyào yuányīn): The main vowel sound
    – Coda (韵尾 yùnwěi): The ending consonant (if present)

The combination of these components is sometimes called the "IMVC" structure
(Initial, Medial, Vowel, Coda).

In some analyses, the final (yùnmǔ) minus the initial consonant is also referred
to as the "rhyme" or "rime" of the syllable.

This syllable structure is particularly important in Chinese because:

• Chinese is a tonal language, and tones are carried on the final part of the
  syllable.
• Many Chinese writing systems and input methods are based on this division
  of syllables.
• It's crucial for understanding Chinese phonology, including rhyme schemes
  in poetry.

This structure is not unique to Chinese but is particularly well-defined
and important in Chinese linguistics due to the language's phonological
characteristics and its writing system.
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- Claude 3.5 Sonnet

$$\text{N, O, P, Q, R, S, T, U, V, W, X, Y, Z}\}$$

$$|\mathbb{L}_E| = 26 = 13 \times 2$$

1.2.3.2. We can classify these letters into consonants and vowels. Because according to some analyses, 'y' can act as either[2], we cannot map each letter to the set of phoneme classes (or 'labels') $\{consonant, vowel\}$. We either have to map it to a set of classes it fulfills, or we can map the classes to the set of letters they apply to or contain, or maybe there is another ideal option, like expressing it as a binary relation. I think that category theory is so useful because it allows us to express such an idea in a way where it would cover all possible variations on the set-theoretic level, so it doesn't matter what set-theoretic convention we use, if we just express this structure categorically instead.

1.2.3.3. My idea is to express the fact that there is a binary relation $R \subset \mathbb{L}_E \times (\text{PhonemeClasses} = \{\text{'Consonant'}, \text{'Vowel'}\})$ by showing that this relation is an object in the category of sets. In order to do this, I think I need to think about what universal properties characterize up to definition a relation, in the category of sets.

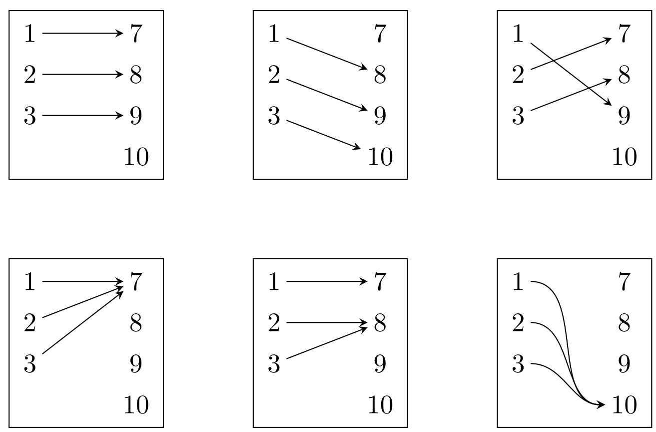
1.2.3.4. A relation is a subset of the product of two sets. That means that it is an element of the set of all subsets of the product of two sets. That means it is an element of the power set of the product of two sets. The power set is isomorphic to 2 to the exponent of A. We could write 2^A as an object in our category, but that's just a label! If we ignore the labels, we realize it is a diagram which defines what "2^A" actually is. We can use the universal property of exponents. I believe we will require a natural numbers object, which gives us the categorical notion of "2". My current simplified way of thinking of the universal property of exponents is that there is an isomorphism between $Hom(A \times B, C)$ and $Hom(A, C^B)$. The idea here is, we want to express that there is an isomorphism between a Hom-set $Hom(X, Y)$, and some object which expresses the set of all functions from X to Y. It might be hard to think how to define this, because they are in "different layers" of the category - a set of arrows, versus an object. It seems like one thing we can do is make them "comparable" or "able to talk to each other" by comparing another Hom-set: the maps between (I think) any other third arbitrary object A, and the maps between $A \times B$ and C. In other words, such an isomorphism allows us to "convert" a question about maps from any object into (or presumably out of, using the dual) an exponential object, into a question about maps from a product into an object (the exponential "base" object).

[2] This may be a red herring. Using IPA and phonemes, rather than the Roman alphabet, we might be able to strictly classify all phonemes as either consonants or vowels. To be researched.

1.2.3.4.1. This reminds me of situations in basic algebra where something seems "unsolvable" until you manipulate it into a form where it becomes solvable. If we want to know what an exponential object "is like", consider what the arrows into the exponential object "are like". If we want to know what the arrows into an exponential object "are like", consider what a Hom-set between any arbitrary third object A and Y^X "is like". If we want to know what the Hom-set $Hom(A, Y^X)$ "is like", the universal property of exponents (I think) says that it "is like" (is very much like - is isomorphic to) the Hom-set $Hom(A \times X, Y)$.

1.2.3.5. Let's think about this for a moment in the category of sets to get a better intuition. Why should $Hom(A \times B, C) \cong Hom(A, C^B)$?

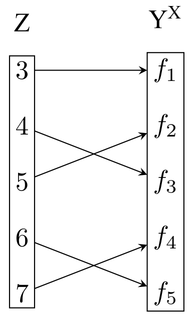
1.2.3.5.1. Let's choose some very simple sets and functions to play with. How about $S1 = \{1, 2, 3\}$, and $S2 = \{7, 8, 9, 10\}$? We can consider all functions from $S1$ to $S2$ (here are just some):



These functions are sets of ordered pairs, like so:

$\{(1,7),$ $(2,8),$ $(3,9)\}$	$\{(1,8),$ $(2,9),$ $(3,10)\}$	$\{(1,9),$ $(2,7),$ $(3,8)\}$
$\{(1,7),$ $(2,7),$ $(3,7)\}$	$\{(1,7),$ $(2,8),$ $(3,8)\}$	$\{(1,10),$ $(2,10),$ $(3,10)\}$

1.2.3.5.2. Now imagine that we choose an arbitrary third set Z, say $\{3, 4, 5, 6, 7\}$, and we consider all functions from Z, to "the set of all functions from X to Y" (which we call Y^X, without really explaining why yet). Our diagram could maybe look like this:



$$f_1 = \{(1,7),(2,8),(3,9)\}$$
$$f_2 = \{(1,8),(2,9),(3,7)\}$$
$$f_3 = \{(1,9),(2,7),(3,8)\}$$
$$f_4 = \{(1,7),(2,7),(3,7)\}$$
$$f_5 = \{(1,8),(2,8),(3,8)\}$$

Which means a given function from Z to Y^X *can* look like this:

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{
  (3, {(1,7),(2,8),(3,9)}),
  (4, {(1,8),(2,9),(3,7)}),
  (5, {(1,9),(2,7),(3,8)}),
  (6, {(1,7),(2,7),(3,7)}),
  (7, {(1,8),(2,8),(3,8)})
}
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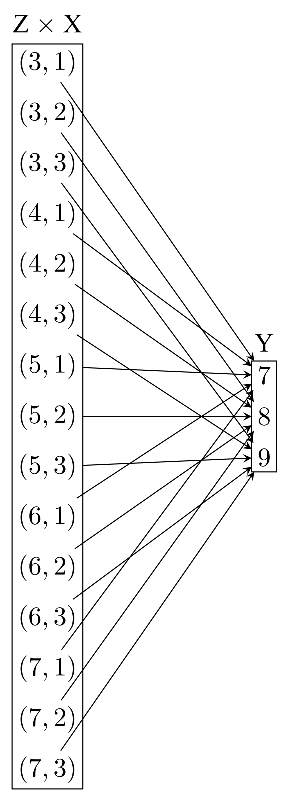
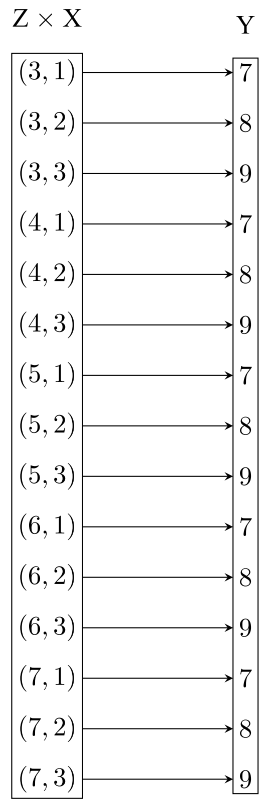
1.2.3.5.3. Now let's consider the case for $Hom(Z \times X, Y)$. Here's an example of what a morphism $f : Z \times X \to Y$ could look like:



Or technically this, too:



A function in Hom(Z × X, Y) can also be represented as:

{
((3,1), 7),
((3,2), 8),
((3,3), 9),
((4,1), 8),
((4,2), 9),
((4,3), 7),
((5,1), 9),
((5,2), 7),
((5,3), 8)
}

(Or this:)

{
((3,1), 7), ((3,2), 8), ((3,3), 9),
((4,1), 7), ((4,2), 8), ((4,3), 9),
((5,1), 7), ((5,2), 8), ((5,3), 9),
((6,1), 7), ((6,2), 8), ((6,3), 9),
((7,1), 7), ((7,2), 8), ((7,3), 9)
}

1.2.3.5.4. I'm going to take a break now, but my guess is, we can shift around the parentheses a little bit and realize how these are the same thing. And I believe it has to do with the concept of 'currying'. It seems like matrices could be used to make this clearer.

Open Challenge: Create a useable computer application that allows people to generate rhymes and slant rhymes for writing raps with. It should use category theory to generate a comprehensive[3] list of English words or phrases that rhyme with a given input, and according to a "degree of rhyme" or a "type of rhyme", beyond "strict rhyme".

[3] It must generate all words and phrases that rhyme according to a given definition of rhyme. It can not return only some.