

## LIST OF CORRECTIONS

### LOCALLY PRESENTABLE AND ACCESSIBLE CATEGORIES

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The following is a list of corrections of all mistakes that have been to our knowledge discovered in our book. The authors are grateful to all who contributed to the list, most of all to D. Benson, M.Hébert, H.-E. Porst and Jan Jürjens.

**Page 3** lines 5-6 (as well as Exercise 1.f(1)): This is incorrect and to be deleted.

**Page 11** line -2:  $C$  should read  $\omega^T$

**Page 14** line 17: all should read: all finite

**Page 16** lines -3 to -1: To verify (2), observe that since  $\mathcal{A}$  is a set and all categories are locally small, see 0.1, all split subobjects of objects of  $\mathcal{A}$  form a set, up to isomorphism. It is sufficient to verify that every finitely presentable object  $K$  is a split subobject...

**Page 21** line 16: 1.14(4) instead of 1.14(3)

**Page 24** line 7 add:  $n \geq 1$

**Page 18** line 3: instead of "since .. contain  $\bar{\mathcal{A}}$ " the text should be:  $\bar{\mathcal{A}}$  is a set because it the the union of the sets  $\mathcal{A}_n, n \in N$ , where

$$\mathcal{A}_0 = \mathcal{A} \cup \{0\} \text{ (for an initial object } 0)$$

and

$$\mathcal{A}_{n+1} = \text{the closure of } \mathcal{A}_n \text{ (under binary coproducts and coequalizers.)}$$

**Page 26** line -8:  $\mathcal{A}^{mor}$  instead  $\mathcal{A}^{obj}$

**Page 30** line 1.33(7): the definition of  $m$  is to be corrected as follows:

$$m : \text{hom}(A_1, -) + \text{hom}(A_2, -) \rightarrow \text{hom}(A_1 \times A_2, -)$$

has components  $\text{hom}(A_i, -) \rightarrow \text{hom}(A_1 \times A_2, -)$  corresponding to the projections.

**Page 31** line 1.33(8): analogously to page 30,

$$m_i : \text{colim}_d \text{hom}(D_i d, -) \rightarrow \text{hom}(\text{lim}_d D_i d, -)$$

The same correction on **Page 40** line 13

**Page 31** line 18 at the end:  $k_{t_0} \cdot f_1 \cdot m_i = k_{t_0,t} \cdot f_2 \cdot m_i$

**Page 31** line 20:  $k_{t_0} \cdot f_1 \cdot m_i = k_{t_0,t} \cdot f_2 \cdot m_i$  which implies, since  $K_t$  is orthogonal to  $m_i$ , that  $k_{t_0,t} \cdot f_1 = k_{t_0,t} \cdot f_2$ .

**Page 33**, line 3: The statement is incorrect: if the construction stops, the object  $X_{i_0}$  is orthogonal. The converse implication is not true in general (consider  $M = \{m\}$  in  $Set$  for an embedding  $m : 1 \rightarrow 1 + 1$ : although 1 is orthogonal, the construction does not stop). However, if in the diagram on p.32 one adds for every pair  $p, q : A' \rightarrow X_i$  the connecting morphism  $m : A \rightarrow A'$  whose domain corresponds to  $f$  defined as  $p.m (= q.m)$ , then the resulting diagram makes the proposition true. This improved construction is used by Max Kelly in [Kelly 1980].

**Pages 34-35**: Theorem 1.39 is true for all uncountable cardinals  $\lambda$ , but the proof presented in our book is wrong. A correct proof can be found in the paper "Uncountable orthogonality is a closure property" by M. Hébert and J. Rosický, Bulletin of the London Mathematical Society 33 (2001), 685-688. For  $\lambda = \omega$ , the implication (ii)  $\implies$  (i) in Theorem 1.39 does not hold, and a more technical description of  $\omega$ -orthogonality classes has been presented in the paper "More on injectivity in locally presentable categories" by M. Hébert, J. Adámek and J. Rosický (Cahiers de Topologie et Geometrie Différentielle et Categoricalue 32(2001), 51-80). A counter-example to 1.39 is also presented in that paper, which is a modification of example by Hugo Volger, see Bibliography (Volger 1979). An independent counter-example has been found by Jan Jürjens in "On a Problem of Gabriel and Ulmer", Journal of Pure and Applied Algebra 158 (2001), 183-196.

**Page 38** line 13:  $d_{A,a} \cdot Ff = d_{A',Hf(a)}$

**Page 38** In (ii) we need to add the argument why is  $Cont_\lambda \mathcal{A}^{op}$  cocomplete. From 1.33(8) we know that it is a  $\lambda$ -orthogonality class in  $Set^{\mathcal{A}^{op}}$ , therefore, it is locally  $\lambda$ -presentable by Theorem 1.39.

**Page 39** lines 10-18: It is not true that the categories  $c_\lambda$ -CAT and  $lp_\lambda$ -CAT are dually equivalent: the correct statement is that the corresponding 2-categories are biequivalent. See J. Adámek and H.-E. Porst: Algebraic theories of quasivarieties, Journal of Algebra 208 (1998), 379-398

**Page 39** line 18:  $Cont_\lambda \mathcal{A}$  instead of  $Cont_\lambda \mathcal{A}^{op}$ , and  $F \mapsto F \cdot H$  instead of  $F \mapsto F \cdot H^{op}$

**Page 40** line 13: see the correction of page 30

**Page 57** line 15: obvious that  $id_A$  does not factor through any  $k_n$

**Page 62** Exercise 1.1: The diagrams of  $S_1 \otimes S_2$  are  $\{A_1\} \times D_2$  and  $D_1 \times \{A_2\}$  where  $A_i$  ranges through all objects of the underlying category of  $S_i$ , and

$D_i$  through all diagrams of  $S_i$ , with

$$\sigma(\{A_1\} \times D_2) = \{id_{A_1}\} \times \sigma_2(D_2)$$

and

$$\sigma(D_1 \times \{A_2\}) = \sigma_1(D_1) \times \{id_{A_2}\}$$

The proof of Proposition 1.53 must be corrected analogously.

**Page 65:** for 1.s see the correction of page 39.

**Page 69 2.3(2):** Scott domains should read: CPOs

**Page 85 line -1:**  $\lambda' \leq \lambda$

**Page 87:** reference in (ii) should be 1.55(1)

**Page 87 line -6:** 1.55(1) instead of 1.55(2)

**Page 88 line -7:**  $b_i : B_i \longrightarrow B (i \leq j)$

**Page 88 line -4:**  $b_i \cdot f_i \cdot \bar{f}_i$  instead of  $f_j \cdot \bar{f}_j \cdot b_{i,j}$

**Page 88 line -1 to Page 89 line 1:**  $b_i \cdot f_i \cdot \bar{f}_i \cdot f_i = b_i \cdot f_i$  (instead of the equations there)

**Page 98 line -5:** left-hand side:  $d_{j_0, j_{0'}}^*$  instead of  $d_{j, j_{0'}}^*$ ; right-hand side:  $d_{j_0, j_{0'}}^*$  instead of  $D^*(j_0 \rightarrow j_{0'})$

**Page 99 line 14 to 15:**  $C_L \downarrow F$  instead of  $F \downarrow C_L$

**Page 101 line -11:** equivalent instead of isomorphic

**Page 109 lines 17-18:** ...whose Kan extension  $F^* : Set^{\mathcal{B}} \rightarrow Set$  has, due to **C**, the property that  $G$  is naturally isomorphic to the domain restriction  $F^*/\mathcal{A}$ . And  $F^*$  preserves  $\lambda$ -small limits of hom-functors since  $G$  does (due to **L**).

**Page 125:** Delete 2.b(2)

**Page 145 line 14:** For infinite  $\lambda > \text{card } \Sigma$

**Page 150 line -12:** This functor is an embedding. In fact, since  $T$  is finitary, it is easy to verify that  $G$  is one-to-one. Moreover, given an algebra  $k : TK \rightarrow K$ , then every element  $x$  of  $TK$  corresponds to an operation symbol  $k'$  in  $\Sigma$  such that  $k(x)$  is the result of  $k'$  in the algebra  $G(K, k)$ . From this it follows easily that  $G$  is full.

**Page 152 line -10:**  $(s(x) = s(y)) \wedge (t(x) = t(y)) \Rightarrow x = y$

**Page 154 line -8:** change reference to example 3.21(1)

**Page 155 in 3.24:** The third sentence of the proof ("It is easy...projective.") is incorrect. The theorem is valid, in fact, it can be strengthened as follows. It suffices to assume the existence of a set  $\mathcal{B}$  of finitely presentable regular projectives such that any object is a regular quotient of a coproduct of objects from  $\mathcal{B}$  (i.e.,  $\mathcal{B}$  is a regular generator). A correct proof of the stronger theorem is as follows:

Conversely, let  $\mathcal{K}$  to a cocomplete category with a regular generator  $\mathcal{B}$  of

finitely presentable regular projectives. It is easy to see that a finite coproduct of regular projectives is a regular projective. Hence  $\mathcal{B}$  can be assumed to be closed under finite coproducts in  $\mathcal{K}$  and, consequently,  $\mathcal{B}^{op}$  can be regarded as an FP sketch. Following Remark 3.17, the category  $\mathcal{V} = Mod_{FP}(\mathcal{B}^{op})$  of all finite-product preserving functors  $\mathcal{B}^{op} \rightarrow Set$  is equivalent to a variety.

Let  $E : \mathcal{K} \rightarrow \mathcal{V}$  be the codomain-restriction of the canonical functor (cf. 1.25). Following 1.26 and 1.27,  $E$  is a faithful right adjoint preserving directed colimits. Since all objects in  $\mathcal{B}$  are regular projectives,  $E$  also preserves regular epimorphisms. We will prove that  $E$  preserves coproducts of objects from  $\mathcal{B}$ . Since  $E$  preserves directed colimits, it suffices to work with finite coproducts. We now simply follow the argument in H.Schubert, *Kategorien*, Akademie-Verlag, Berlin 1970 (see 10.3.5 there):

Let

$$B_1 \xrightarrow{i_1} B_1 + B_2 \xleftarrow{i_2} B_2$$

be a coproduct in  $\mathcal{B}$ , and

$$EB_1 \xrightarrow{l_1} EB_1 + EB_2 \xleftarrow{l_2} EB_2$$

be a coproduct in  $\mathcal{V}$ . Let

$$f : EB_1 + EB_2 \longrightarrow E(B_1 + B_2)$$

be the induced morphism. By the argument from the implication (i)  $\Rightarrow$  (ii) in the proof of 1.46, we get a morphism  $g : E(B_1 + B_2) \longrightarrow EB_1 + EB_2$  satisfying  $g \cdot E i_k = l_k$ ,  $k = 1, 2$ . We have  $g \cdot f = id_{EB_1 + EB_2}$  and, since  $E$  is full on  $\mathcal{B}$ ,  $f \cdot g = id_{E(B_1 + B_2)}$ . Therefore  $E(B_1 + B_2) \cong EB_1 + EB_2$ .

Next we prove that  $E$  is full. Consider a morphism  $f : EK \longrightarrow EK'$  in  $\mathcal{V}$ . There exists a regular epimorphism  $p : \coprod_{j \in J} B_j \longrightarrow K$  where  $w_i : B_i \longrightarrow \coprod_{j \in J} B_j$  is a coproduct of objects from  $\mathcal{B}$ . There are  $h_j : P_j \longrightarrow K'$  such that

$$f \cdot Ep \cdot Ew_j = Eh_j$$

for any  $j \in J$ . Since  $E$  preserves coproducts of  $\mathcal{B}$ -objects, we get  $h : \coprod_{j \in J} B_j \longrightarrow K'$  with  $f \cdot Ep = Eh$ .

Let

$$L \xrightleftharpoons[v]{u} \coprod_{j \in J} B_j \xrightarrow{p} K$$

be a kernel pair. Then

$$EL \xrightleftharpoons[E v]{E u} E \coprod_{j \in J} B_j \xrightarrow{E p} EK$$

is a kernel pair. Since

$$Eh \cdot Eu = f \cdot Ep \cdot Eu = f \cdot Ep \cdot Ev = Eh \cdot Ev,$$

we get  $hu = hv$  and, therefore, we obtain a morphism  $t : K \rightarrow K'$  with  $t \cdot p = h$ . Hence  $Et \cdot Ep = Eh = f \cdot Ep$ , thus  $f = Et$ .

We have proved that  $\mathcal{K}$  is equivalent to a full reflective subcategory of  $\mathcal{V}$  closed under directed colimits. Following Theorem 3.22, it remains to prove that  $E(\mathcal{K})$  is epireflective in  $\mathcal{V}$ . Any  $A$  in  $\mathcal{V}$  can be presented by a coequalizer

$$E \coprod_{j \in J} B_j \begin{array}{c} \xrightarrow{Eu} \\ \xrightarrow{Ev} \end{array} E \coprod_{i \in I} C_i \xrightarrow{p} A$$

in  $\mathcal{V}$  where  $B_j, C_i$  belong to  $\mathcal{B}$  for  $i \in I$  and  $j \in J$ . Then a reflection  $r_A : A \rightarrow EA^*$  of  $A$  to  $E(\mathcal{K})$  is given by a coequalizer

$$\coprod_{j \in J} B_j \begin{array}{c} \xrightarrow{u} \\ \xrightarrow{v} \end{array} \coprod_{i \in I} C_i \xrightarrow{q} A^*$$

in  $\mathcal{K}$  via  $r_A \cdot p = Eq$ . Since  $Eq$  is a regular epimorphism in  $\mathcal{V}$  and  $\mathcal{V}$  is a variety,  $r_A$  is a regular epimorphism in  $\mathcal{V}$ .

**Page 157:** Delete "colimit of... is full" from lines 6 to 7. Insert: To prove that  $H$  is full, let  $p : HK_1 \rightarrow HK_2$  be a homomorphism. The canonical diagram of  $K_1$  w.r.t.  $\mathcal{B}$  has the following compatible cocone:

$$\text{to } f : P_s \rightarrow K_1 \text{ assign } (U^*p)_s(f) : P_s \rightarrow K_2.$$

The compatibility, i.e.,  $(U^*p)_s(f) \cdot h = (U^*p)_{s'}(f \cdot h)$  for all  $h : P_{s'} \rightarrow P_s$ , follows from the commutation of  $p$  with the  $\Sigma$ -operation  $\sigma = \sigma_{X_s, h} : s \rightarrow s'$ : we have

$$(U^*p)_{s'}(f \cdot h) = [(U^*p)_s \cdot \sigma_{HK_1}](f) = [\sigma_{HK_2}(U^*p)_s](f) = (U^*p)$$

Since  $\mathcal{B}$  is dense, there exists a morphism  $q : K_1 \rightarrow K_2$  with  $(U^*p)_s(f) = qf$  for all  $f$ , thus  $(U^*p)_s = (Uq)_s = (U^*Hq)$ , which proves  $p = Hq$ .

**Page 159** line 16 Add:

$$(x_0 \vee x_1 \vee x_2 \vee \dots) \vee y_1 \vee y_2 \vee \dots = x_0 \vee y_1 \vee x_1 \vee y_2 \vee x_2 \vee \dots$$

**Page 163** line 12:  $(s(x) = s(y)) \wedge (t(x) = t(y)) \Rightarrow x = y$

**Page 163** line 18: Def  $(\sigma) = \{s(x) = s(y), t(x) = t(y)\}$

**Page 163** line -14:  $t(x)$  instead of  $t(s)$

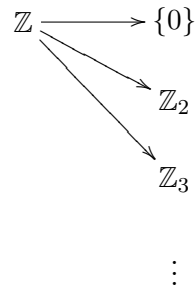
**Pages 164 to 167:** The proof of Theorem 3.36 is not correct. A correct proof is presented in J.Adámek, M.Hébert and J.Rosický: "On essentially

algebraic theories and their generalizations”, Algebra Universalis 41 (1999) 213-227.

**Page 179** line -6: is weakly

**Page 180:** 4.10(3) is wrong

**Page 184:** In 4.15 the cone should be changed to the cone of *all* torsion quotients of  $\mathbb{Z}$ , i.e.:



**Page 190** Example: (1) From "Given ... " on the text should be changed as follows:

Given a strictly ordered poset  $(X, <)$ , a multireflection is formed by all canonical maps  $c : (X, <) \rightarrow (X/\sim, \sqsubset)$  where

1.  $\sim$  is an equivalence relation on  $X$  such that the derived relation  $<^*$  on  $X/\sim$  (with  $M <^* N$  iff  $m < n$  for some  $m \in M, n \in N$ ) is irreflexive

and

2.  $\sqsubset$  is a linear order extending  $<^*$ .

There is a missing arrow in the concrete example: the canonical arrow w.r.t. the equivalence with classes  $\{a\}$  and  $\{b, c\}$ .

**Page 192, Theorem 4.29:** This theorem is wrong. See our Correction of pages 34-35: Given a reflective subcategory  $\mathcal{A}$  of a locally finitely presentable category  $\mathcal{K}$  closed under directed colimits, then if  $\mathcal{A}$  is not an  $\omega$ -orthogonality class, then it is not an  $\omega$ -cone-orthogonality-class either. In fact, given a cone to which all objects of  $\mathcal{A}$  are orthogonal, then there exists a member of that cone to which, again, all objects of  $\mathcal{A}$  are orthogonal (this is an easy consequence of the fact that  $\mathcal{A}$  is closed under products of pairs in  $\mathcal{K}$ ). Thus, a presentation of  $\mathcal{A}$  as a cone-orthogonality class reduces to a presentation as an orthogonality class.

**Page 195** line -6 and -3:  $Set^A$  should read  $Set^S$

**Page 197** line -6: regular epimorphisms instead of monomorphisms

**Page 197** line -1 should read: projective w.r.t. every epimorphism with kernel  $A$  iff  $\text{Ext}(G, A) = 0$

**Page 198** line 4: multicolimits

**Page 201** lines -7 to -6:  $A$  has less than  $\lambda$  edges, i.e.,  $\text{card} \bigcup_{\sigma \in \Sigma_{rel}} \sigma_A < \lambda$  (instead of the equations there)

**Page 207** in 5.8(3): the last axiom should be replaced by the following:

$(\forall x_1, \dots, x_6)(\text{comp}(x_1, x_2, x_3) \wedge \text{comp}(x_2, x_4, x_6) \wedge \text{comp}(x_3, x_4, x_5) \Rightarrow \text{comp}(x_1, x_6, x_5))$ ,

and

$(\forall x_1, \dots, x_6)(\text{comp}(x_1, x_2, x_3) \wedge \text{comp}(x_2, x_4, x_6) \wedge \text{comp}(x_1, x_6, x_5) \Rightarrow \text{comp}(x_3, x_4, x_5))$

**Page 208** line 8: 2.46 should read 2.48

**Page 210** line 15:  $\text{id}_X : (X, \emptyset) \rightarrow (X, X \times X)$

**Page 213**, second line under 5.17(3): of subsets of  $I$  such that the intersection of any finite subcollection is non-empty, there exists...

**Page 238** line 6: instead of "positive-primitive" write "conjunctions of atomic formulas"

**Page 264** line 8: accessible iff it satisfies

**Page 297**: Problem 9, this is a repetition of Problem 5