Symmetric Monoidal Categories with Attributes

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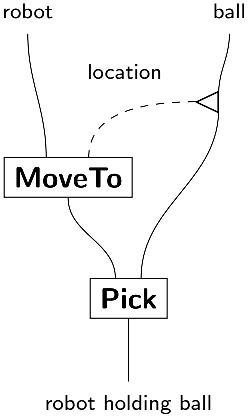
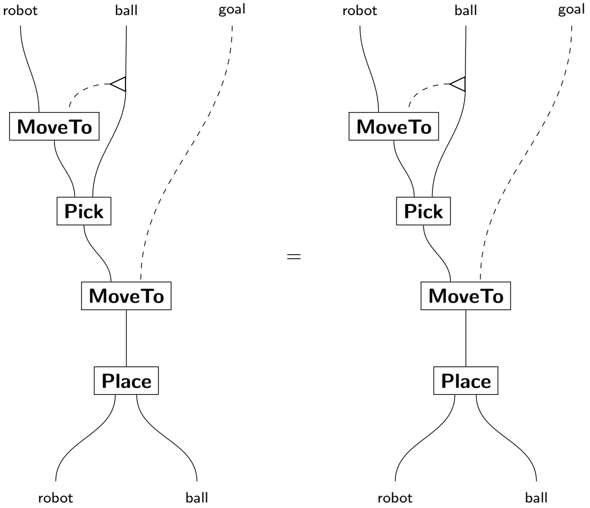
What does CT bring to robotics?

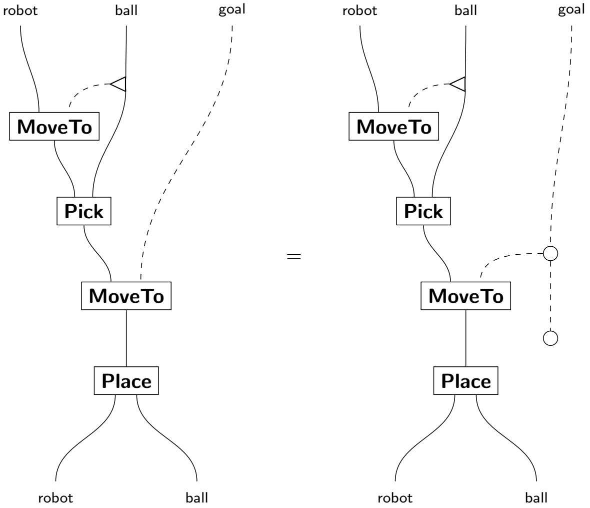
- [ACR] gives some motivation
  - Robot programming involves interactions between several domains
  - String diagram calculus is intuitive, helps non-experts



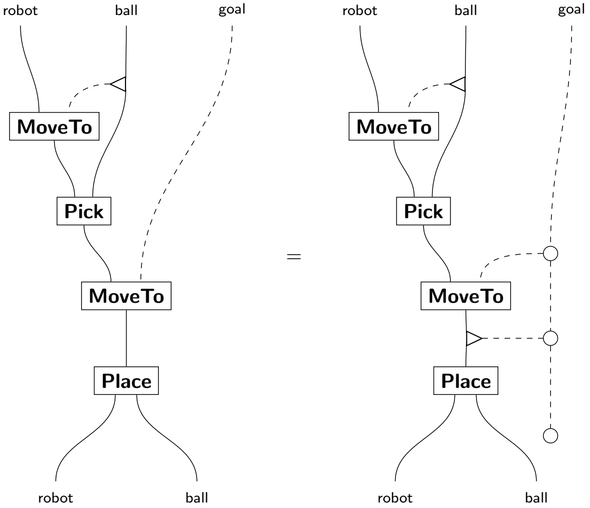
Diagrammatic proofs



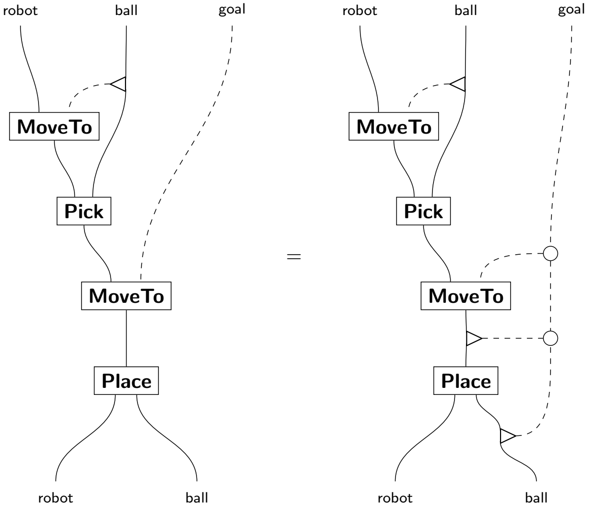
Diagrammatic proofs



Diagrammatic proofs

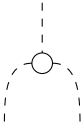
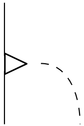


Diagrammatic proofs



Questions to resolve

What makes different from ?

What are  and  ? How do they behave?

Data services

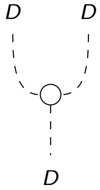
- Introduced by [Pav13]
- Axiomatize "common ways to process data"

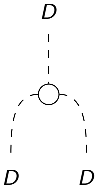
Definition

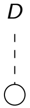
A data service in a SMC $\mathcal{C}$ is an object D of $\mathcal{C}$ together with morphisms

- Multiplication $\mu : D \otimes D \to D$ (intuition: filter for equality)
- Comultiplication $\delta : D \to D \otimes D$ (intuition: copy)
- Counit $\epsilon : D \to I$ (intuition: delete)

satisfying SCFA-like axioms.

$\mu =$ 

; $\delta =$ 

; $\epsilon =$ 

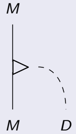
Attributes

- Attach information to objects through *actions*

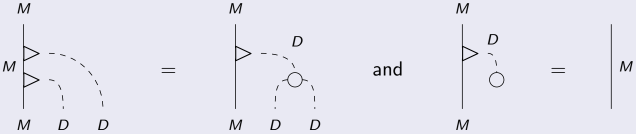
Definition

Let $\mathcal{C}$ be a SMC and D be a data service in $\mathcal{C}$. A (right) *data service action* of D on $M \in \mathrm{Ob}(\mathcal{C})$ consists of a (right) comonoid action $\gamma : M \to M \otimes D$. We depict this γ as:

$\gamma =$

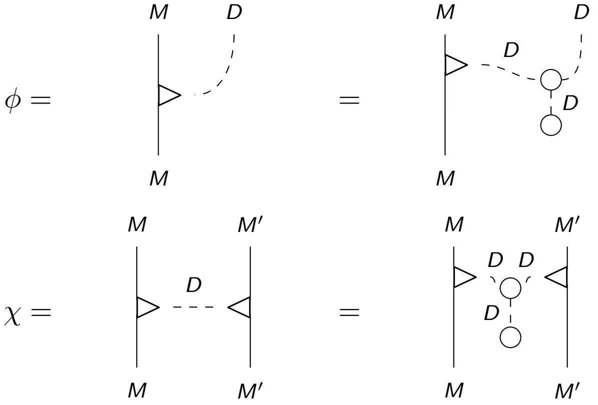


The comonoid action axioms are:



Filtering *entities*

Given such a γ we can define morphisms that filter entities for equality:



Putting everything together

Definition

Let $\mathcal{C}$ be a SMC and $U : \mathbf{Data}(\mathcal{C}) \to \mathcal{C}$ be the forgetful functor. An *attribute structure* on $\mathcal{C}$ consists of:

- A category $\mathcal{A}$, called the *category of attributes*
- A functor $E : \mathcal{A} \to \mathcal{C}$
- A functor $V : \mathcal{A} \to \mathbf{Data}(\mathcal{C})$
- A natural transformation $\gamma : E \to E \otimes (U \circ V)$

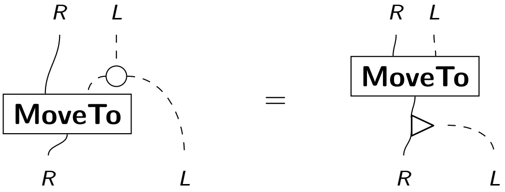
such that:

- For every $A \in \mathrm{Ob}(\mathcal{A})$, γ_A gives a comonoid action of $V(A)$ on $E(A)$.

(When we have such a structure in mind, we call $\mathcal{C}$ a *category with attributes*.)

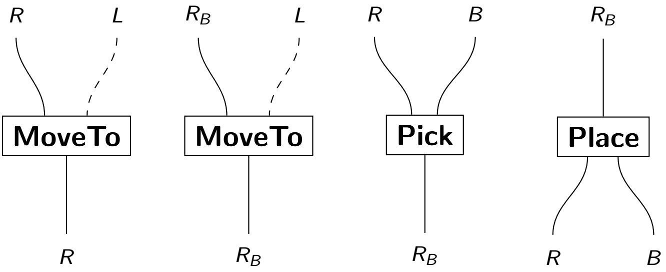
Naturality of actions

- Processes in the real world have effects on attributes
- Morphisms should let us predict these effects
- This can be formalized as a *naturality condition* on the γ maps



An example

- SMC generated by:
  - Objects: location L, robot R, ball B, robot holding ball R_B
  - Morphisms: data service structure on L, and:

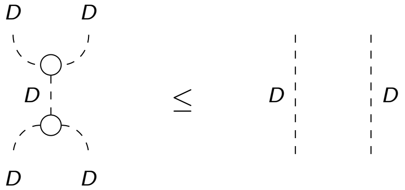


- Attribute structures let us impose structured equations

Poset-enrichment

- Key example: **PartFn** ($f \leq g$ iff g extends f)
- **Poset**-enrichment reflects (relative) *partial definition*
- How to implement:
  - Make all categories / functors involved **Poset**-enriched
  - Impose extra conditions on data services / morphisms (see [BPS17])

e.g.



Semantics

- Two types considered so far:
  - **Boolean** - Planning Domain Definition Language
  - **Geometric** - Canonical Robot Command Language
- Typical workflow:
  - Create a PDDL problem description
  - Convert PDDL plans into string diagrams
  - Compile the diagrams into CRCL

Takeaways

- String diagrams can clarify robotics planning
- CT can connect different engineering semantics
- Research into this area is still developing!
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References

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Filippo Bonchi, Dusko Pavlovic, and Paweł Sobociński. "Functorial semantics for relational theories". In: *arXiv preprint arXiv:1711.08699* (2017).

Dusko Pavlovic. "Monoidal computer I: Basic computability by string diagrams". In: *Information and computation* 226 (2013), pp. 94–116. DOI: 10.1016/j.ic.2013.03.007.