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ERRATUM TO "FROM LOOP GROUPS TO 2-GROUPS"

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Abstract

There were a number of sign errors in our paper "From loop groups to 2-groups" [*Homology Homotopy Appl.* **9** (2007), 101–135]. Here we explain how to correct those errors.

The following sign corrections to our paper [**BCSS**] make the article consistent with [**BC**] and [**LM**], in particular the definition of morphisms between 2-term L_∞-algebras. The definition given in [**BC**], Definition 4.3.4, was independently checked to be consistent with the one suggested in [**LM**, Remark 5.3] by Kevin van Helden. Since the main theorem of the paper involves several such morphisms, a number of signs in the data of some L_∞-algebra morphisms need to be changed. Moreover, the definition of the crossed module action α in Proposition 2.4 led to an inconsistency independent of the L_∞-algebra material, found by David Michael Roberts. The corrected sign is self-consistent, as well as agreeing with the rest of the paper. There were additionally some typos in an earlier paper [**MS**] leading to some innocuous sign errors that do not impact the other calculations.

All the corrections here have been made in the current arXiv version of our paper [**BCSS**]. All of our calculations have been independently checked by Rist, Saemann and Wolf [**RSW**], as well as by Roberts.

- The big commutative diagram in Definition 2.2 should be replaced by

$$
\begin{array}{ccc}
[[F_0(x),F_0(y)],F_0(z)] & \xrightarrow{J_{F_0(x),F_0(y),F_0(z)}} & [F_0(x),[F_0(y),F_0(z)]] + [[F_0(x),F_0(z)],F_0(y)] \\
\downarrow{\scriptstyle [F_2,1]} & & \downarrow{\scriptstyle [1,F_2]+[F_2,1]} \\
[F_0[x,y],F_0(z)] & & [F_0(x),F_0[y,z]] + [F_0[x,z],F_0(y)] \\
\downarrow{\scriptstyle F_2} & & \downarrow{\scriptstyle F_2+F_2} \\
F_0[[x,y],z] & \xrightarrow{F(J_{x,y,z})} & F_0[x,[y,z]] + F_0[[x,z],y]
\end{array}
$$

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- Equation (5) needs to be replaced by

$$\begin{aligned}\phi_1(l_3(x,y,z)) - l_3(\phi_0(x),\phi_0(y),\phi_0(z)) = \\ \phi_2(x,l_2(y,z)) + \phi_2(y,l_2(z,x)) + \phi_2(z,l_2(x,y)) + \\ l_2(\phi_0(x),\phi_2(y,z)) + l_2(\phi_0(y),\phi_2(z,x)) + l_2(\phi_0(z),\phi_2(x,y))\end{aligned}$$

- The definition of κ in equation (12) should be

$$\kappa(f,g) = \exp\left(-2ik\int_0^{2\pi}\int_0^{2\pi}\langle f(t)^{-1}f'(t), g'(\theta)g(\theta)^{-1}\rangle\, d\theta\, dt\right),$$

correcting a typo in [**MS**].

- The definition of the normal subgroup N below equation (12) should read

  Let N be the subset of $P_0\Omega G \times U(1)$ consisting of pairs (γ, z) such that $\gamma\colon [0,2\pi] \to \Omega G$ is a loop based at $1 \in \Omega G$ and

$$z = \exp\left(ik\int_{D_\gamma}\omega\right)$$

  where D_γ is any disk in ΩG with γ as its boundary.

  for consistency with the definition of κ above.

- The definition of $d\alpha$ in the statement of Proposition 3.1 should be

$$d\alpha(p)(\ell,c) = \big([p,\ell],\ 2k\int_0^{2\pi}\langle \ell(\theta), p'(\theta)\rangle\, d\theta\,\big).$$

- The definition of β_p in the proof of Proposition 3.1 should be

$$\beta_p(\xi) = 2\int_0^{2\pi}\langle \xi(\theta), p(\theta)^{-1}p'(\theta)\rangle\, d\theta.$$

- The definition of ϕ_2 in the statement of Lemma 5.4 should be

$$\phi_2(p_1,p_2) = k\int_0^{2\pi}\left(\langle p_2, p_1'\rangle - \langle p_2', p_1\rangle\right)\, d\theta$$

- The definition of λ_2 in the proof of Lemma 5.5 should be

$$\lambda_2(\ell_1,\ell_2) = \big(0, 2k\int_0^{2\pi}\langle \ell_1, \ell_2'\rangle\, d\theta\big)$$

With these changes, the calculations in the proofs all go through, leaving the results unchanged.

References

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