In the sequence $2, 4, 16, \ldots$, it looks like each term is the square of the one before, so I guess that b_3 is $16^2 = 256$. If $b_{n+1} = b_n^2$, I believe that means that b_n is "b_0 squared n times". This can be written as $b_0^{(2^n)}$.

This makes me think that we can show how to construct terms by "squaring" the set of terms from the previous round. I explored some options on paper, but haven't found a definitive pattern yet.

For elements $\{0, 1\}(= B(0))$, there seems to be a correspondence between the "square" of that set and the set $\{0, 1, a, \neg a\}$. For example:

	0	1
0	$(0,0)$	$(1,0)$
1	$(0,1)$	$(1,1)$

versus

	0	1
0	0	1
1	a	$\neg a$

It feels like the binary values in the product are "controllers" or "selectors" which can generate the terms in the second table. We could interpret the first element of each ordered pair in the second row of the first table as "choosing whether a will have a negation symbol or not". $(0, 1)$ says "no negation symbol", $(1, 1)$ says "yes, negation symbol".

For b_1, with generator a, we would have:

	0	1	a	$\neg a$
0	$(0,0)$	$(1,0)$	$(a,0)$	$(\neg a,0)$
1	$(0,1)$	$(1,1)$	$(a,1)$	$(\neg a,1)$
a	$(0,a)$	$(1,a)$	(a,a)	$(\neg a,a)$
$\neg a$	$(0,\neg a)$	$(1,\neg a)$	$(a,\neg a)$	$(\neg a,\neg a)$

versus

	0	1	a	
0	0	1	a	
1	b	$\neg b$	$a \wedge b$	a
a	$a \wedge \neg b$	$a \vee \neg b$	$\neg a \wedge b$	$\neg$
$\neg a$	$\neg a \wedge \neg b$	$\neg a \vee \neg b$	$(a \wedge b) \vee (\neg a \wedge \neg b)$	$(a \wedge \neg b)$

I've been trying to find a way to order the above table so that the pattern is more evident and shows some kind of symmetry. Here are some of my thoughts:

One way to group the terms could be:

- 0, 1 (2 elements)
- a, $\neg a$, b, $\neg b$ (4 elements)
- $a \wedge b$, $a \vee b$, $a \wedge \neg b$, $a \vee \neg b$, $\neg a \wedge b$, $\neg a \vee b$, $\neg a \wedge \neg b$, $\neg a \vee \neg b$ (8 elements)
- $(a \wedge b) \vee (\neg a \wedge \neg b)$, $(a \wedge \neg b) \vee (\neg a \wedge b)$ (2 elements)

This grouping suggests an arithmetic progression:

$$2, 4, 8, 2$$

and for the next round:

$$2, 4, 8, 16, \ldots$$

summing to 256.

After considering various arrangements, I struggled with the observation that the diagonals of a square are 1, 2, 3, 4, 3, 2, 1, but I often did not find any of the above elements that fit that naturally into groups of 3. There may be some, but when I chose some, I often found it odd to place the other elements somewhere.

But I've come up with an ordering that seems to make a bit more sense:

$$\begin{array}{cccc} a \wedge b & a \vee b & a & 0 = (a \wedge \neg a) \\ a \wedge \neg b & a \vee \neg b & \neg a & 1 = (a \vee \neg a) \\ \neg a \wedge b & \neg a \vee b & b & (a \wedge b) \vee (\neg a \wedge \neg b) \\ \neg a \wedge \neg b & \neg a \vee \neg b & \neg b & (a \wedge \neg b) \vee (\neg a \wedge b) \end{array}$$

This arrangement seems to capture some of the symmetry and relationships between the terms.

This makes me think that when you are having difficulty finding a formula to generate terms in a sequence, it might at least be easier to find an *ordering* for those terms. I think for the above, some simple rules like "the negation of something should come before you move to the next variable" can help you at least put things in a sensible order. But this is just a hypothesis.

I will probably need a very small hint or I will try to "zoom out" a bit and rely on my usual way of asking conceptual questions to understand what's "really going on".

After writing this, I saw an idea more clearly:

AND	**OR**	**Selection**	**Symmetrical Operations?**
$a \wedge b$	$a \vee b$	"select left" $\equiv a$	$\bot \equiv (a \wedge \neg a) \equiv 0$
$a \wedge \neg b$	$a \vee \neg b$	"select left and negate" $\equiv \neg a$	$\top \equiv (a \vee \neg a) \equiv 1$
$\neg a \wedge b$	$\neg a \vee b$	"select right" $\equiv b$	a equals $b \equiv a \leftrightarrow b \equiv (a \wedge b) \vee (\neg a \wedge \neg b)$
$\neg a \wedge \neg b$	$\neg a \vee \neg b$	"select right and negate" $\equiv \neg b$	a XOR $b \equiv a \oplus b \equiv (a \wedge \neg b) \vee (\neg a \wedge b)$

0 and 1 are logical operations, sort of like the most agreeable vs. disagreeable person in the world, who no matter what you tell them tell you it's true / false.

Even though this appears to be the number of possible Boolean operations on 2 elements, it still leaves the questions why it is the "square" of the previous round. I'll probably explore $n = 3$ as a way to go deeper. I also want to think about why Boolean operations on a collection of n formula are the same thing as n-ary Boolean operations. It feels like the answer will be quite simple, when I see it, but I need to "see" it, first. My guess is it has something to do with formulae being mappable to their possible truth values $(0, 1)$.