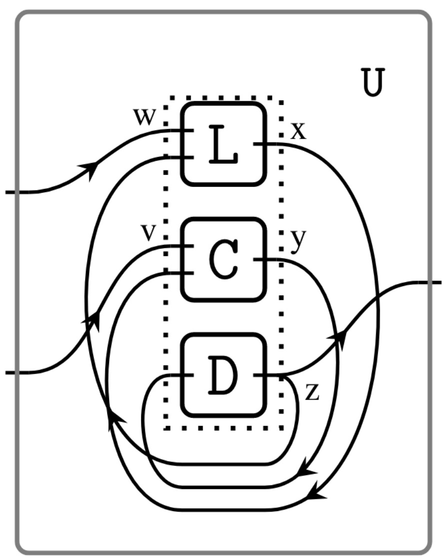
Compositional cyber-physical systems modeling



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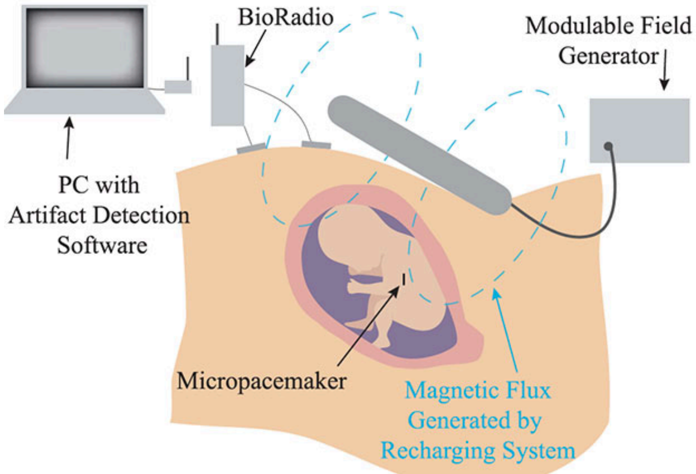
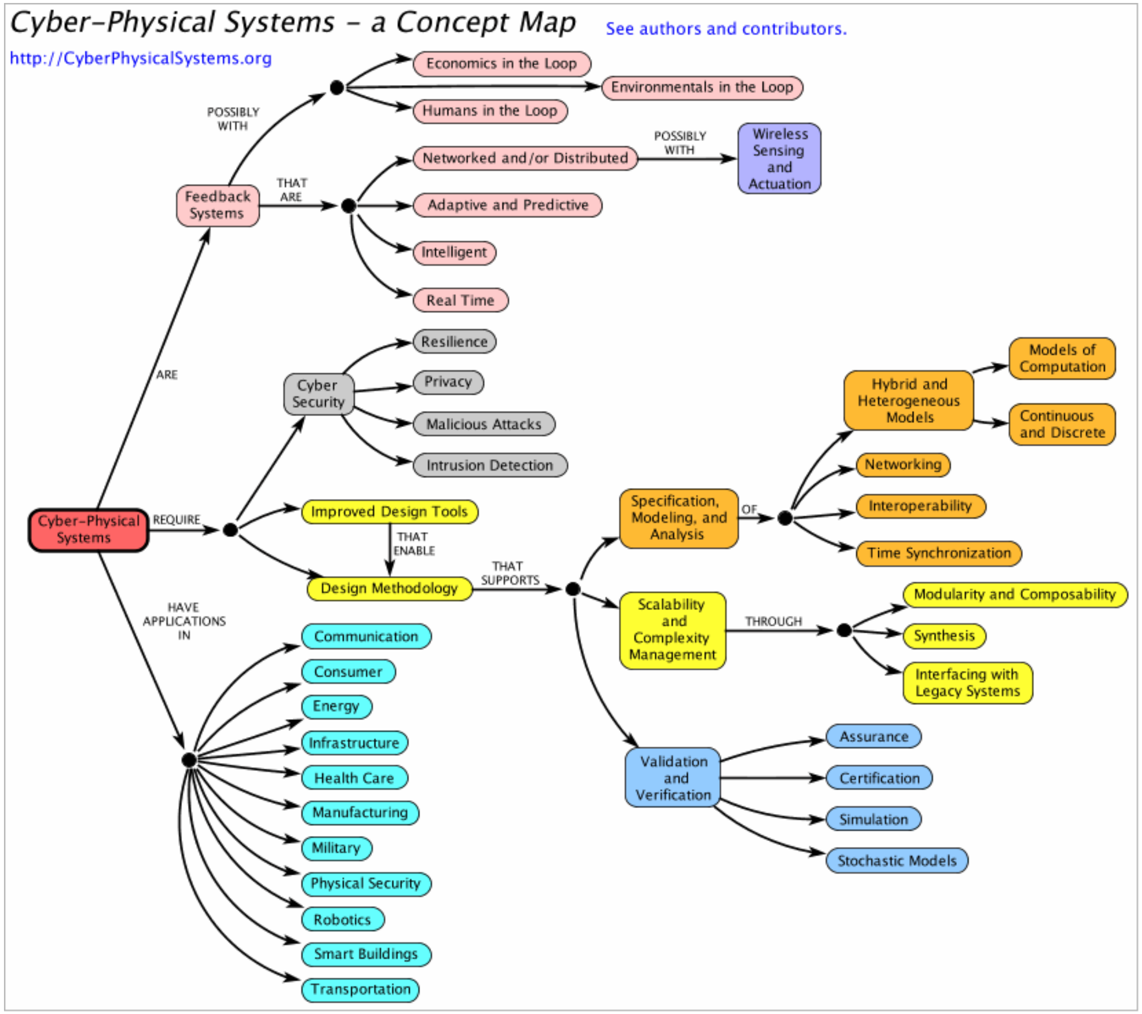
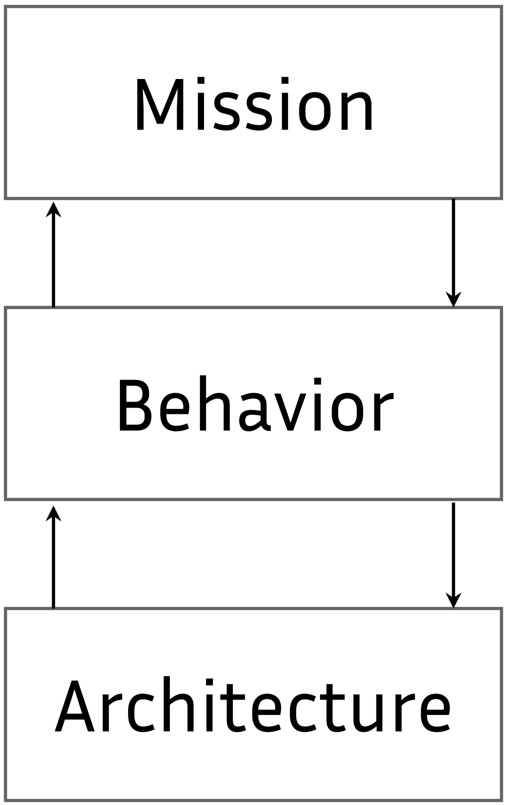
Cyber-physical systems are ubiquitous



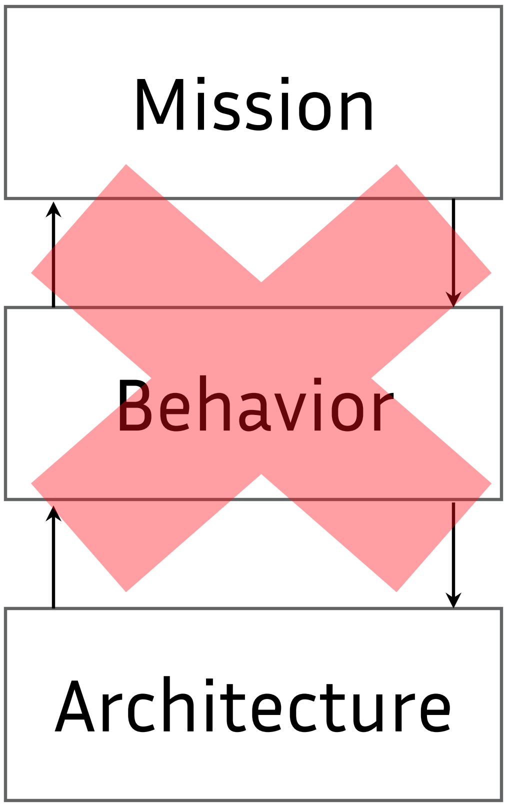
Each leaf node has its own formalism but relations are largely informal



Designing cyber-physical systems requires tracing different model views



But what are the traces?



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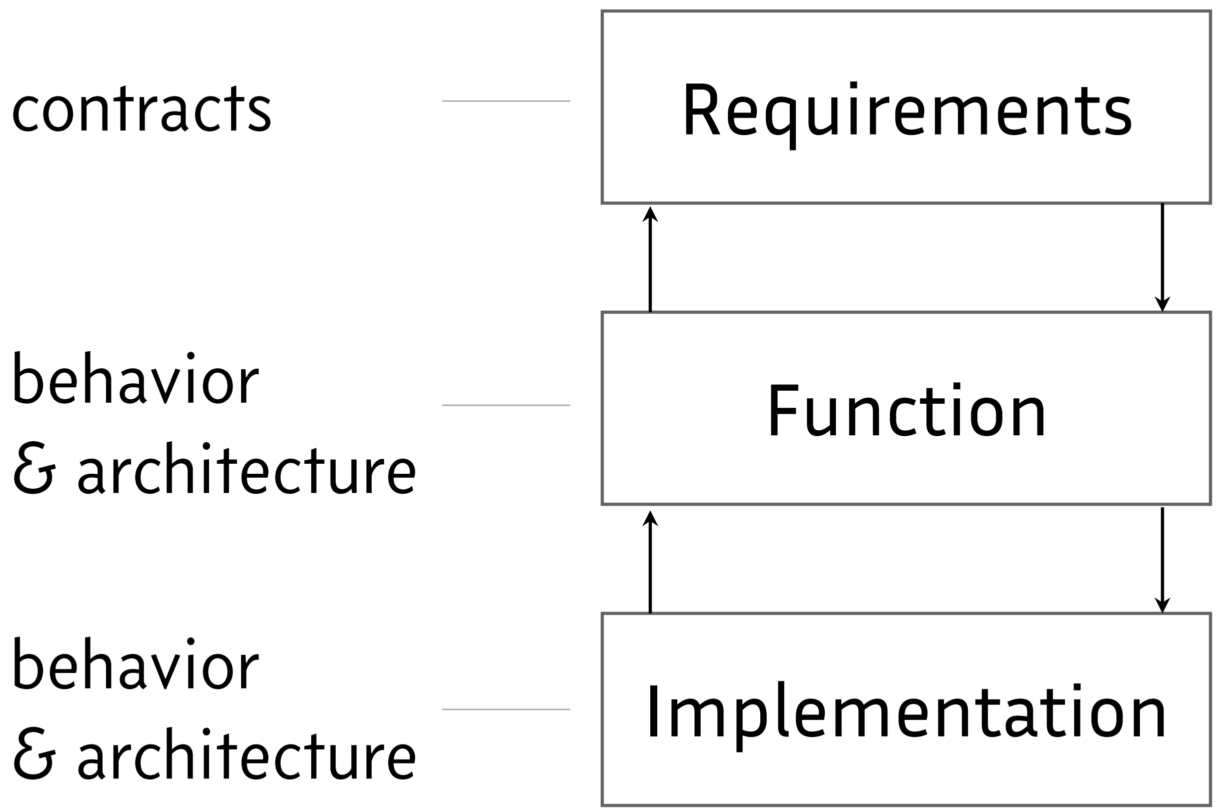
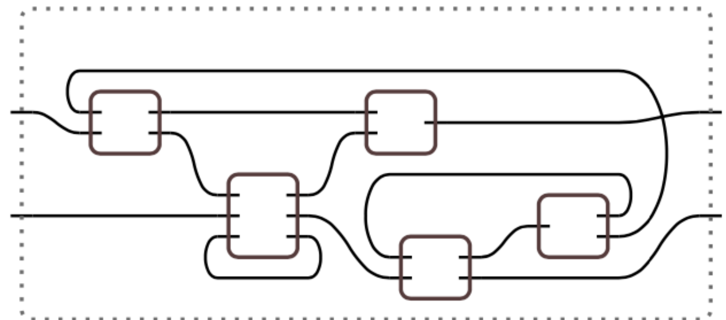


Our aim is to build a metatheory for *compositional* system theory



To combat:

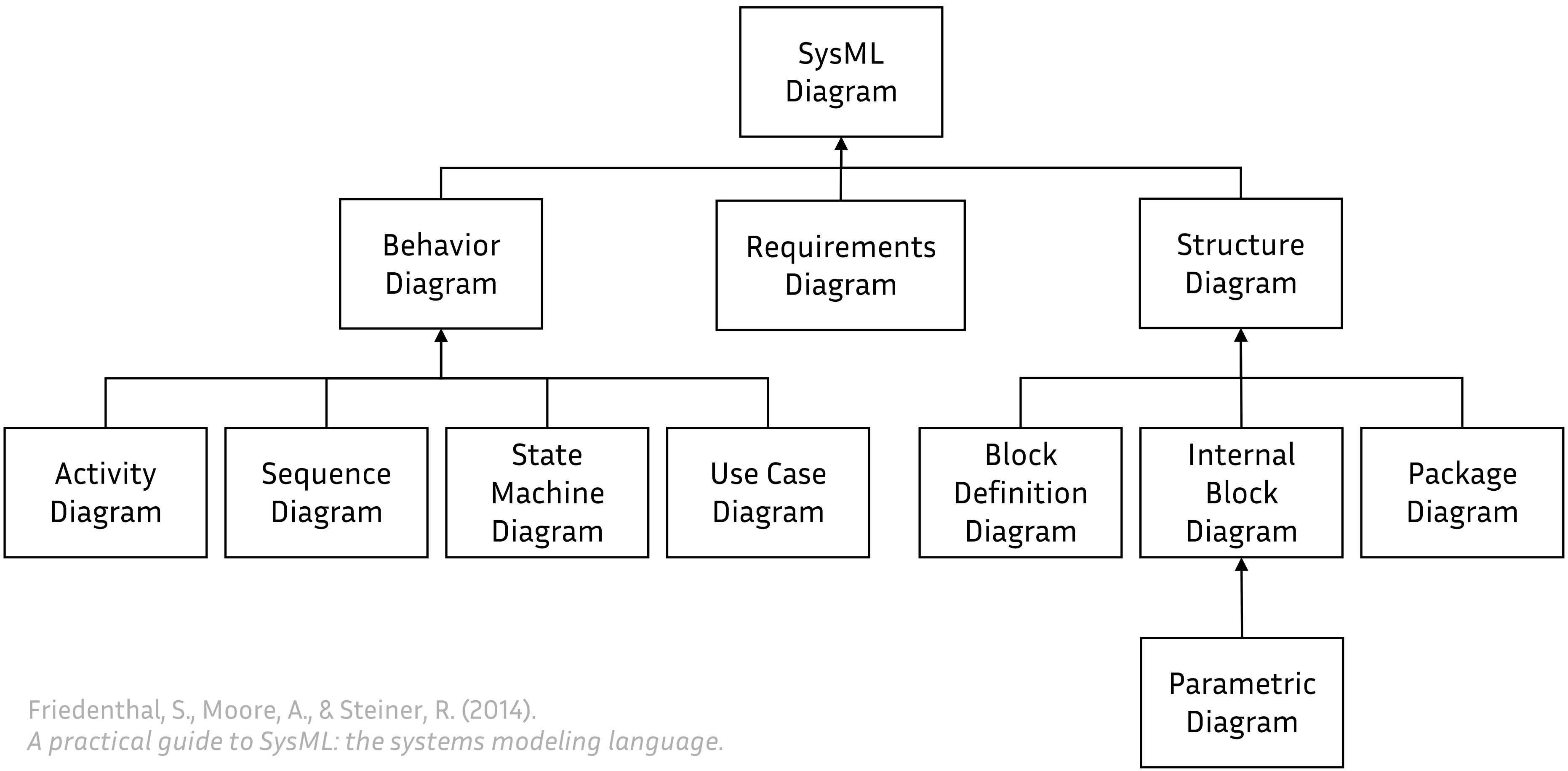
- Semantic ambiguity
- Complexity of *models*
- Intractability between different model views

Category theory leads to unification and scalability

Formal representation of processes and interactions (dynamic view)

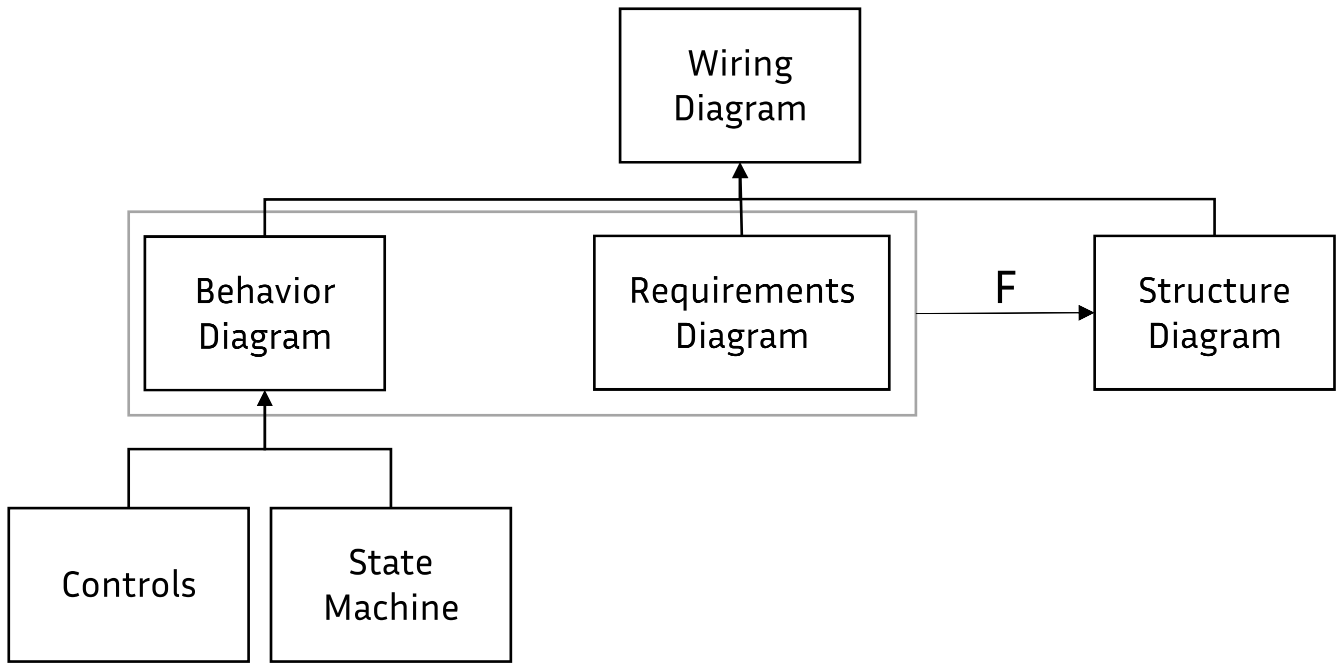
Relational ontologies (static view)

Putting the applied in category theory requires us to use it as a language



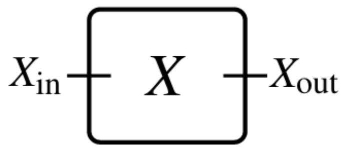
Friedenthal, S., Moore, A., & Steiner, R. (2014).
A practical guide to SysML: the systems modeling language.

Wiring diagrams bridge behaviors, requirements, and architectures

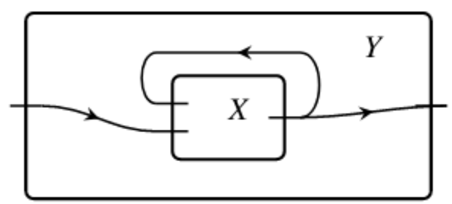


Wiring diagrams formalize processes and interactions

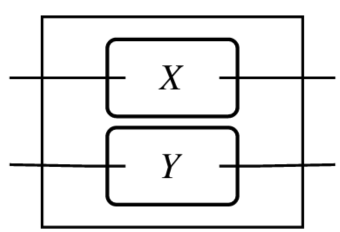
Objects are pairs of sets $X = (X_{in}, X_{out})$



Morphisms are functions $(X_{out} \times Y_{in} \xrightarrow{\varphi_{in}} X_{in}, X_{out} \xrightarrow{\varphi_{out}} Y_{out})$

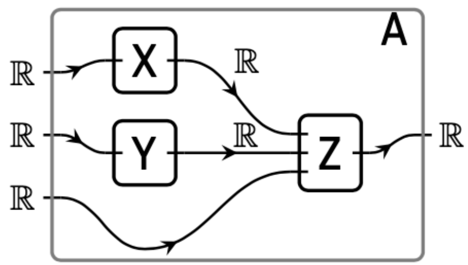


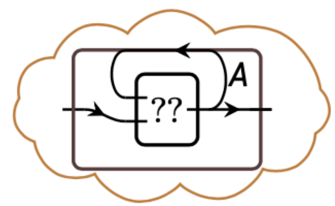
Has a tensor product $X \otimes Y = (X_{in} \times Y_{in}, X_{out} \times Y_{out})$



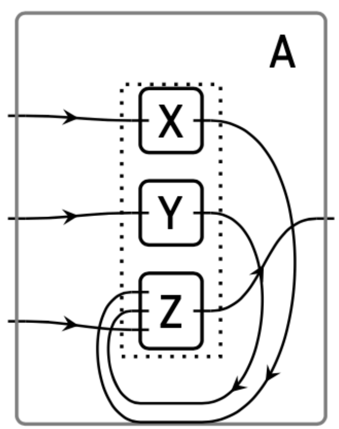
Wiring diagrams model systems as algebras

Start with three boxes $\mathbb{R} - \boxed{X} - \mathbb{R}$, $\mathbb{R} - \boxed{Y} - \mathbb{R}$ and $\mathbb{R},\mathbb{R},\mathbb{R} - \boxed{Z} - \mathbb{R}$ where all input and output data of possible processes are real numbers, interconnected as in





This is a morphism $X \otimes Y \otimes Z \to A$ in the category **WD** expressed as



$$\begin{cases} \phi_{\mathrm{in}}: \overbrace{\mathbb{R}\times\mathbb{R}\times\mathbb{R}}^{(X\otimes Y\otimes Z)_{\mathrm{out}}} \times \overbrace{\mathbb{R}\times\mathbb{R}\times\mathbb{R}}^{A_{\mathrm{in}}} \xrightarrow{\pi_{12456}} \overbrace{\mathbb{R}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}}^{(X\otimes Y\otimes Z)_{\mathrm{in}}} \\ \phi_{\mathrm{out}}: \underbrace{\mathbb{R}\times\mathbb{R}\times\mathbb{R}}_{(X\otimes Y\otimes Z)_{\mathrm{out}}} \xrightarrow{\pi_3} \underbrace{\mathbb{R}}_{A_{\mathrm{out}}} \end{cases}$$

Wiring diagrams address emergence

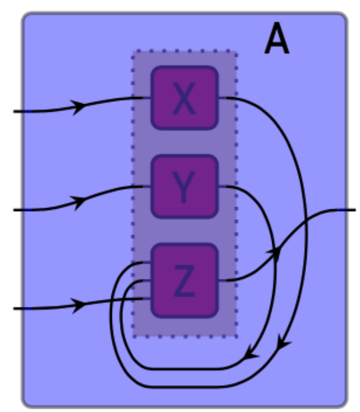
A $\mathbf{WD}$-algebra, namely a monoidal functor

$$\begin{array}{ccc} F\colon \mathbf{WD} & \longrightarrow & \mathbf{Cat} \\ X{=}(X_{\text{in}}, X_{\text{out}}) & \longmapsto & FX \\ \phi \downarrow & & \downarrow F\phi \\ Y{=}(Y_{\text{in}}, Y_{\text{out}}) & \longmapsto & FY \end{array}$$

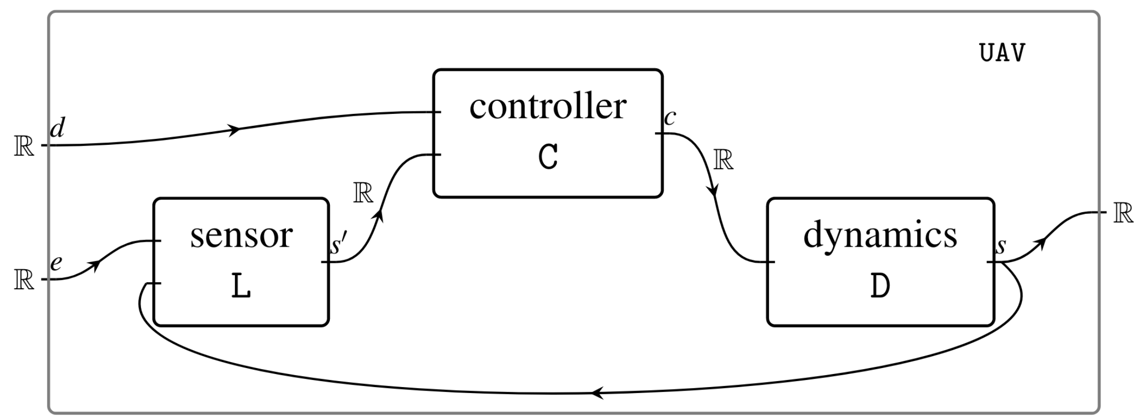
subsystems category

composite system functor

gives *semantics* to boxes, *composite operation* to wiring diagrams and *parallelizing operation* to subsystems via $FX \times FY \to F(X \otimes Y)$



Wiring diagrams allow us to formalize and relate each of these model views



$$f_{\mathrm{in}}\colon \mathbb{R}^3 \times \mathbb{R}^2 \to \mathbb{R}^5,\ (s', c, s, e, d) \mapsto (e, s, d, s', c)$$
$$f_{\mathrm{out}}\colon \mathbb{R}^3 \to \mathbb{R},\ (s', c, s) \mapsto s$$

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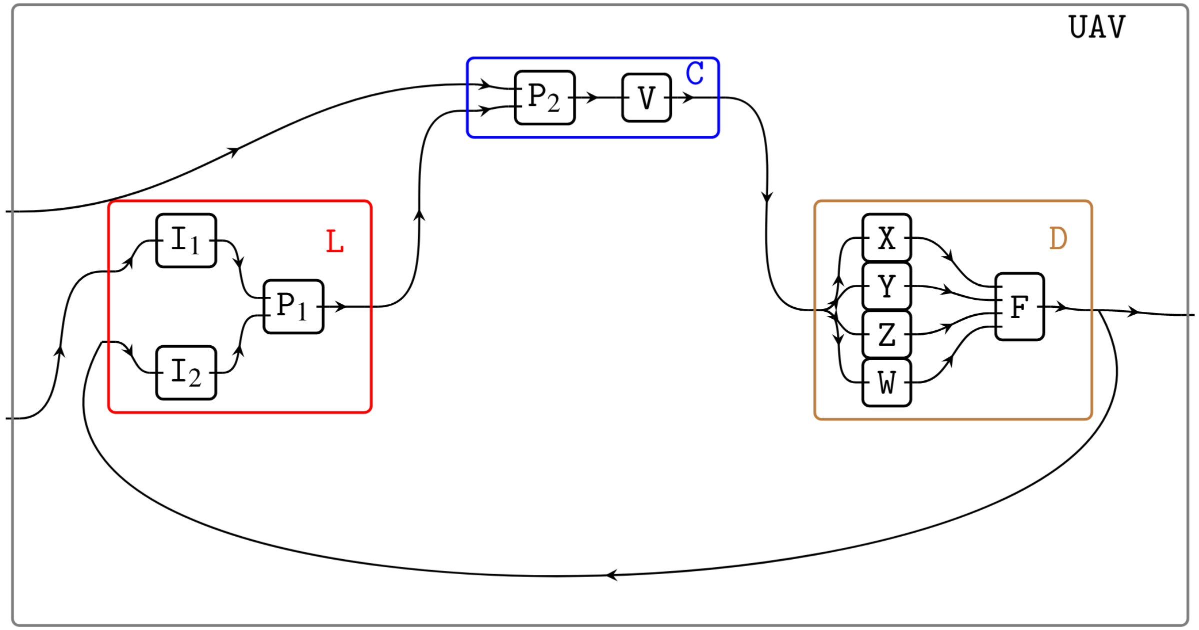
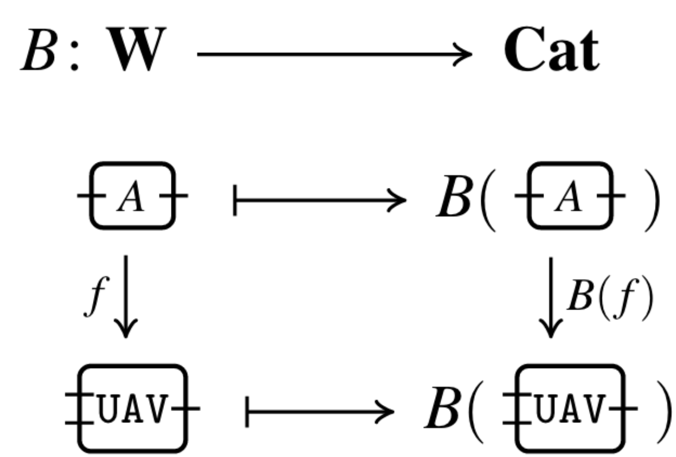
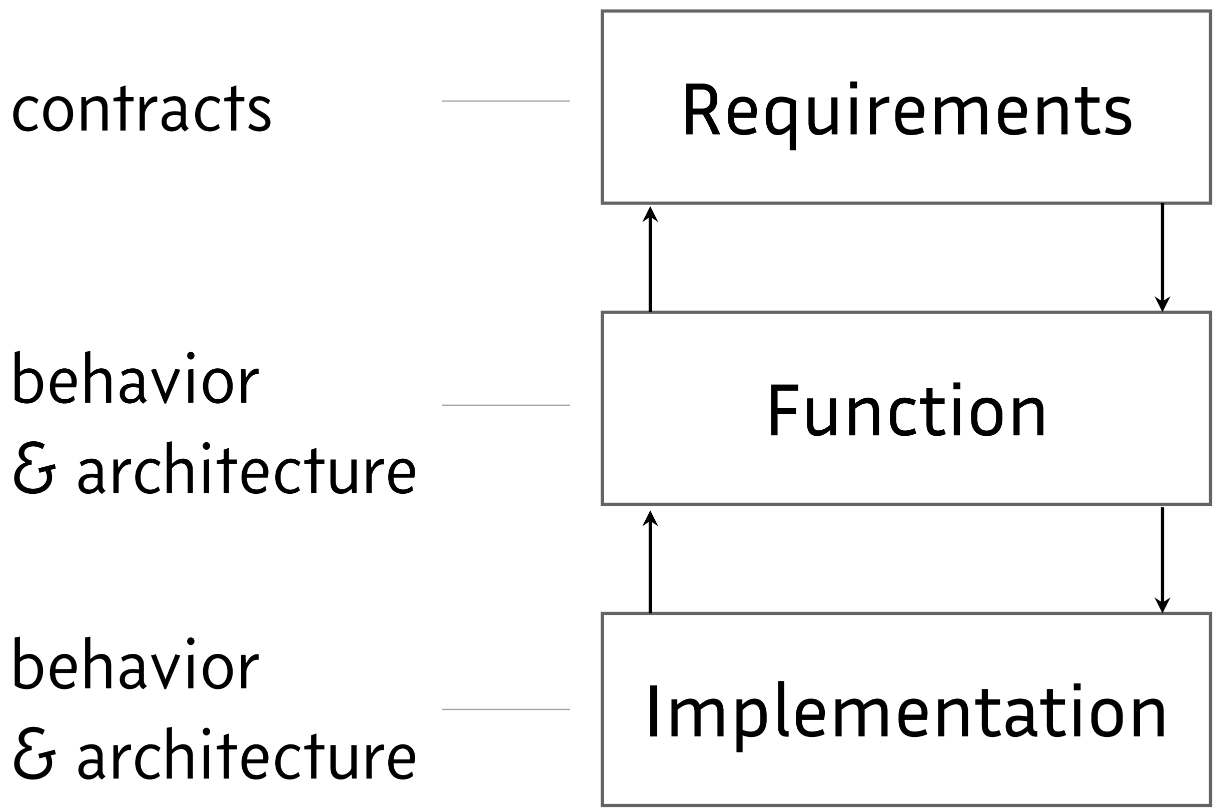


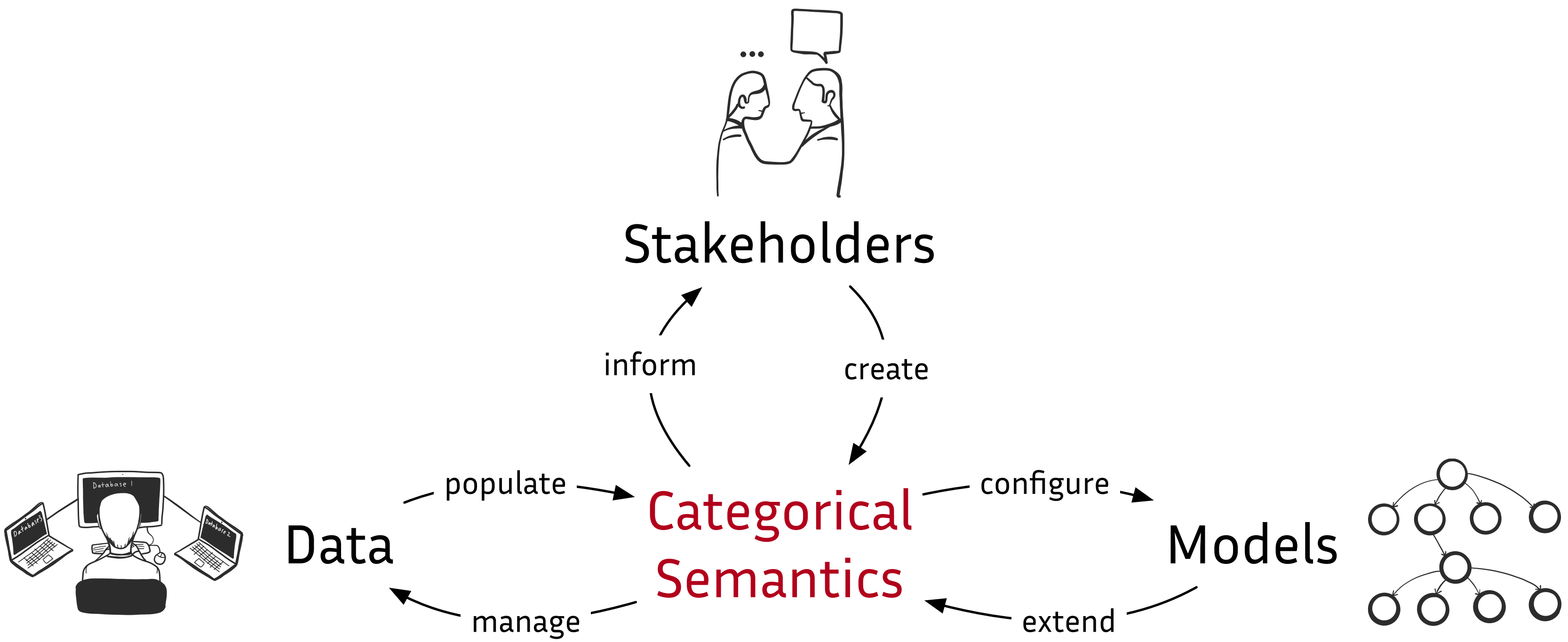
The insight that we formally capture with category theory is the *meaning* of the traces between model views



A categorical formulation allows for percolating constraints



Category theory has the potential to unify and scale system models



The future of safety and security coengineering

(Step 1) Produce a contracts algebra that captures a subset of safety requirements

(Step 2) Model attacker actions over the systems model

⇒ *Unify* safety and security models for cyber-physical systems

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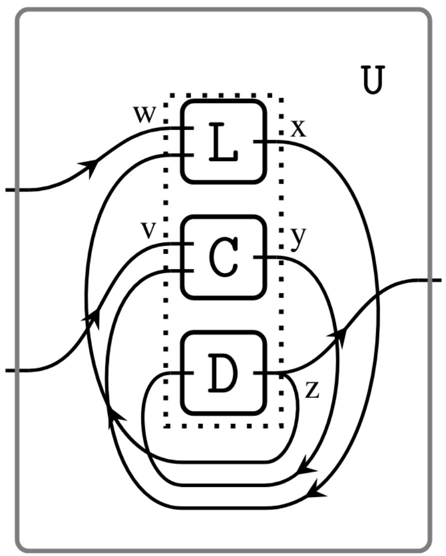

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