A Diagrammatic Approach To Symmetric Lenses

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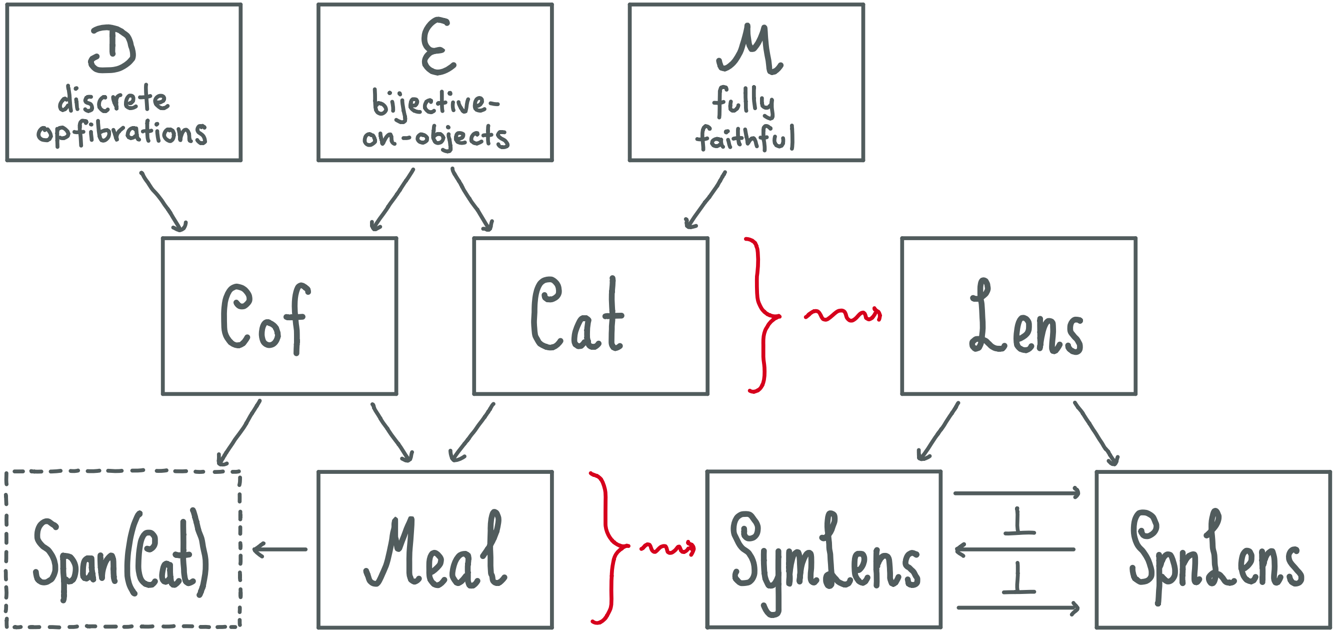
APPLIED CATEGORY THEORY 2020

Overview of the talk

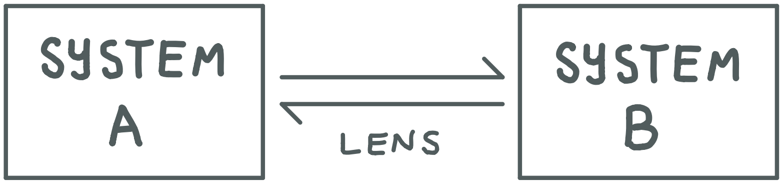


GOAL 1: Develop a diagrammatic framework for lenses.

GOAL 2: Understand the relationship between symmetric & asymmetric lenses.

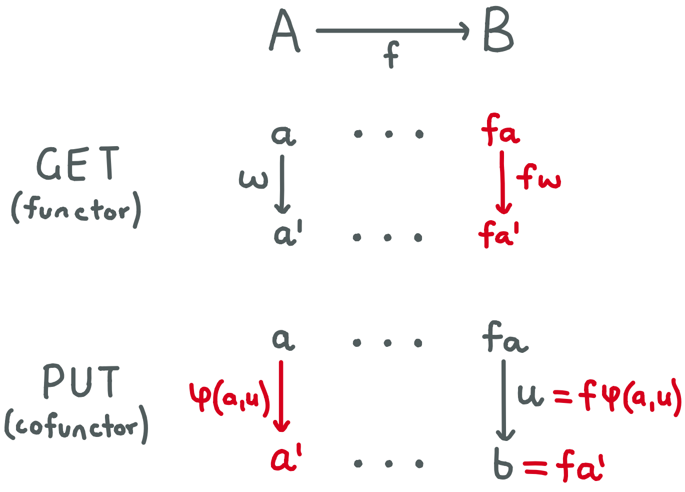
What is a lens?

CATEGORY
objects = states
morphisms = updates

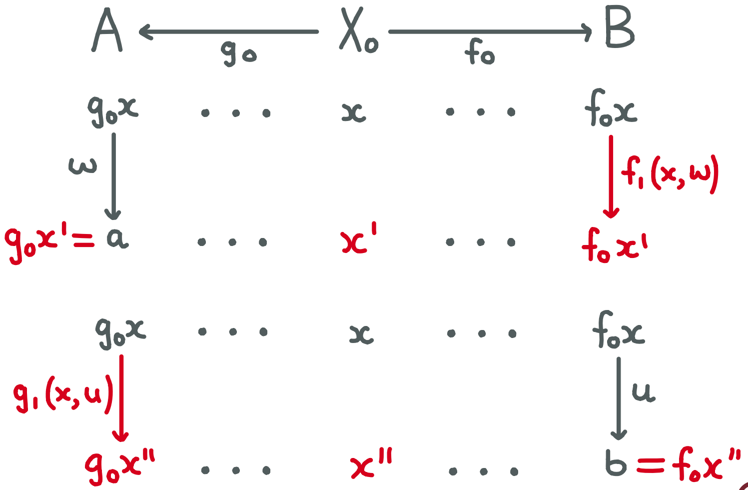


"maintains consistency between states of systems"

ASYMMETRIC LENS



SYMMETRIC LENS

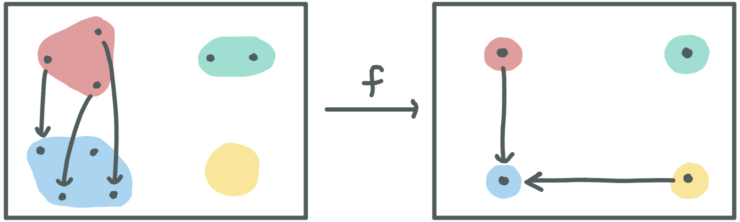


Three classes of functors

$\mathcal{D}$ — discrete opfibrations

$$A \xrightarrow{f} B$$

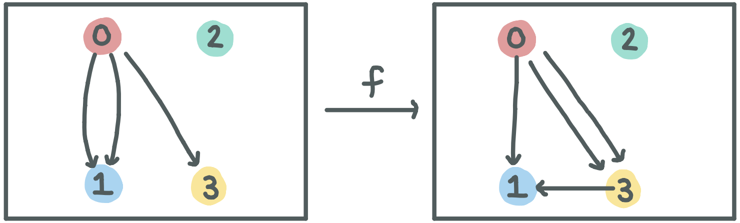
$$\begin{array}{ccc} a & \cdots & fa \\ \exists! w \downarrow & & \downarrow u = fw \\ a' & \cdots & b = fa' \end{array}$$



$\mathcal{E}$ — bijective-on-objects

$$A \xrightarrow{f} B$$

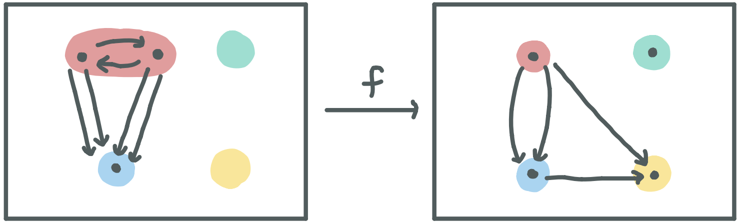
$$\exists!\ a \quad \cdots \quad b = fa$$



$\mathcal{M}$ — fully faithful

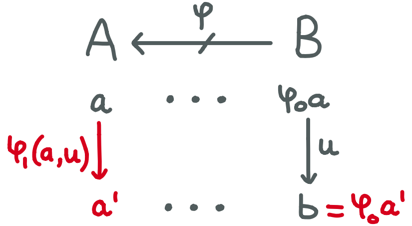
$$A \xrightarrow{f} B$$

$$\begin{array}{ccc} a & \cdots & fa \\ \exists! w \downarrow & & \downarrow u = fw \\ a' & \cdots & fa' \end{array}$$



Cofunctors & factorisation systems

Cof
small categories & cofunctors



"each update $u : \varphi_0 a \to b \in B$ has a chosen lift"

+

respects identities and composition

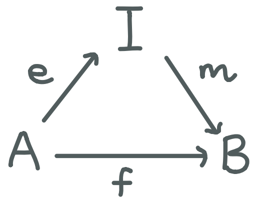
Cat
small categories & functors

factorisation system ⇝

$\mathcal{E}$
bijective-on-objects

+

$\mathcal{M}$
fully faithful



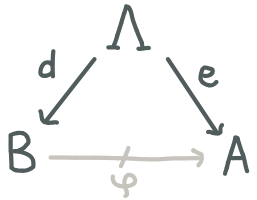
Cof
small categories & cofunctors

factorisation system ⇝

$\mathcal{D}^{op}$
discrete opfibrations

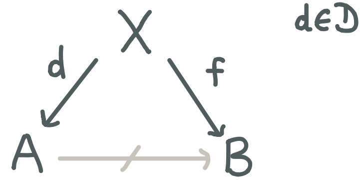
+

$\mathcal{E}$
bijective-on-objects

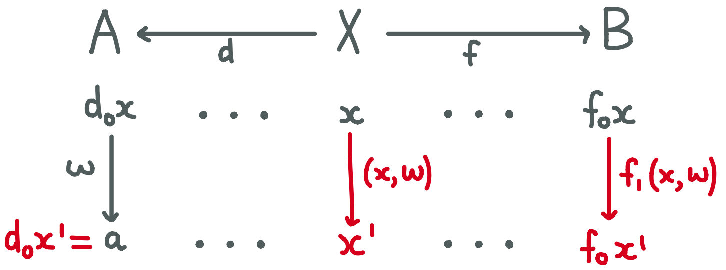


THE BICATEGORY OF MEALY MORPHISMS

Meal
small categories, Mealy mor. & 2-cells



"partial map between categories"

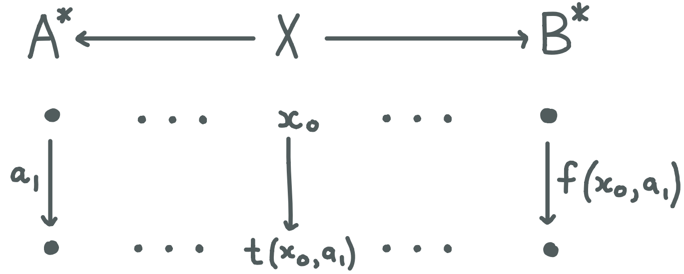


EXAMPLE: MEALY MACHINES

$f : X \times A \longrightarrow B$ OUTPUT

$t : X \times A \longrightarrow X$ TRANSITION

"Mealy morphism between free monoids"



Meal (small categories, Mealy mor. & 2-cells) $\simeq$ Mnd(Span) — factorisation system ⇝ $\mathcal{D}^{op}$ (discrete opfibrations) + $\mathcal{E}$ (bijective-on-objects) + $\mathcal{M}$ (fully faithful)

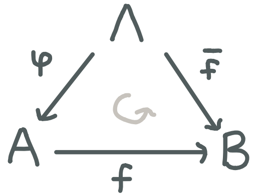
Cof: $\mathcal{D}^{op}$ + $\mathcal{E}$

Cat: $\mathcal{E}$ + $\mathcal{M}$

Asymmetric lenses

Cof + **Cat** ⇝ **Lens** (Small categories & asym. lenses)

"PUT" "GET"



$\varphi \in \mathcal{E}$

$\bar{f} \in \mathcal{D}$

"functor with a suitable choice of lifts"

Examples

- A, B codiscrete ⇝ very well-behaved lenses

  $f : A \longrightarrow B$ $\qquad p : A \times B \longrightarrow A$

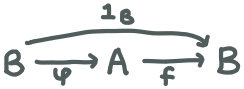
  (PUT-GET) $fp(a,b) = b$

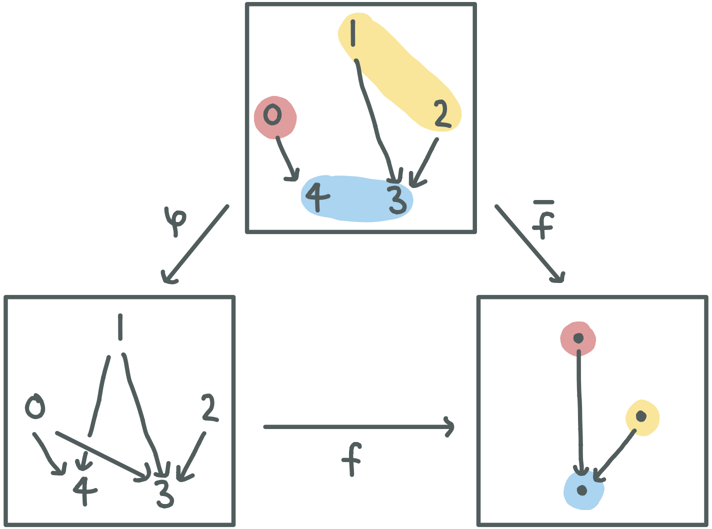
  (GET-PUT) $p(a, fa) = a$

  (PUT-PUT) $p(p(a,b), b') = p(a, b')$

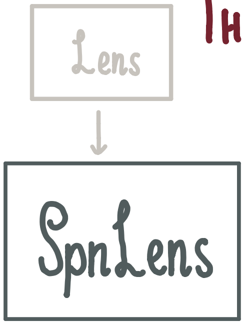
  } LENS LAWS

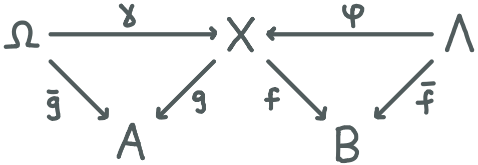
- Split opfibrations ⇝ $B \longrightarrow \mathcal{C}at$
- A, B monoids ⇝ section/retraction



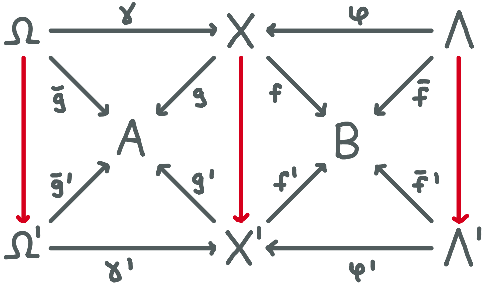


The bicategory of spans of asymmetric lenses

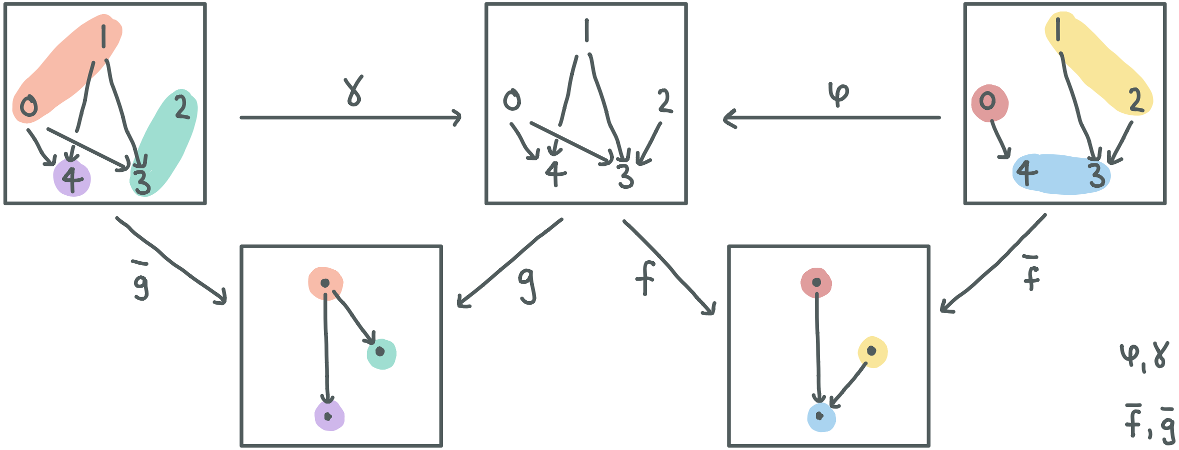




1-cells



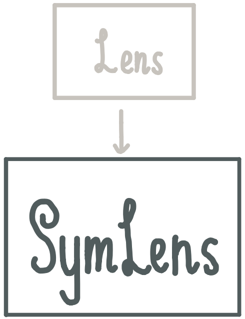
2-cells



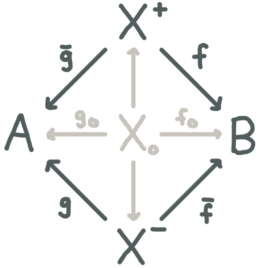
$\varphi, \gamma \in \mathcal{E}$

$\bar{f}, \bar{g} \in \mathcal{D}$

The bicategory of symmetric lenses

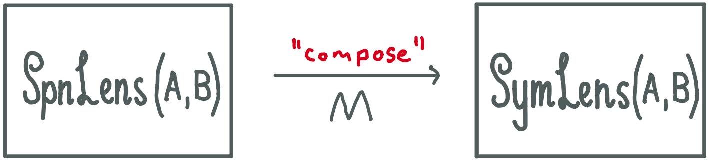


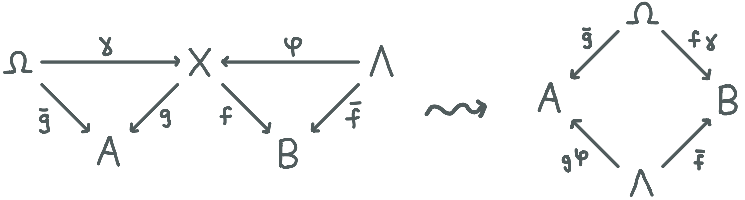
"suitable pair of Mealy morphisms"

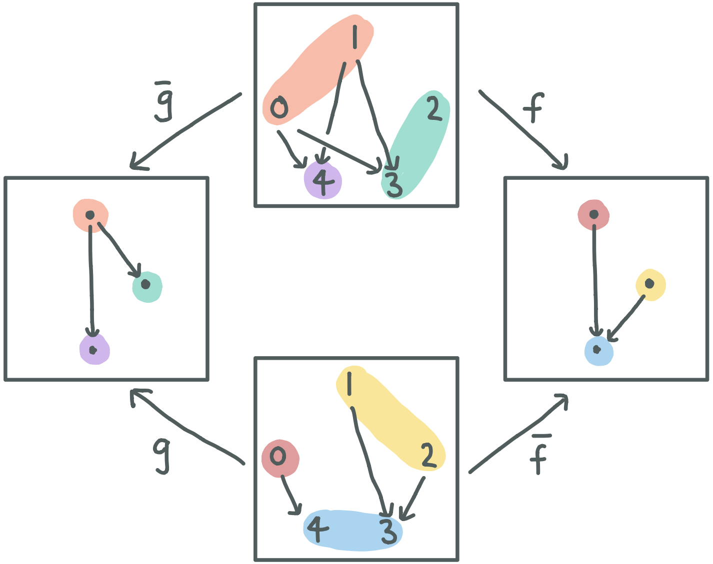


1-cells

$\bar{g}, \bar{f} \in \mathcal{D}$



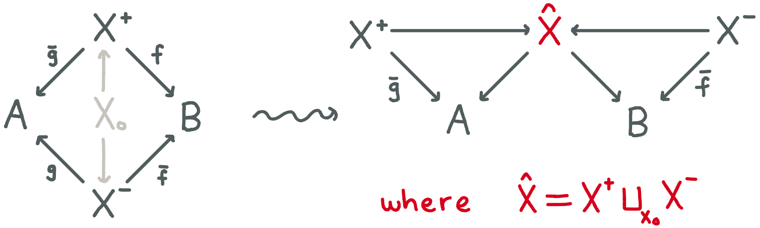




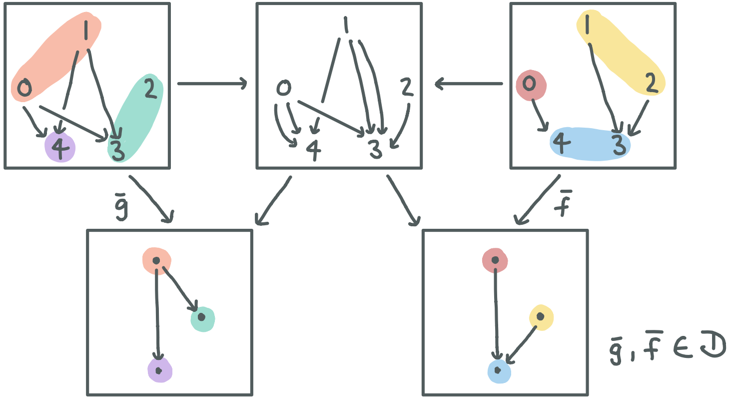
$\bar{g}, \bar{f} \in \mathcal{D}$

Main theorem: an adjoint triple

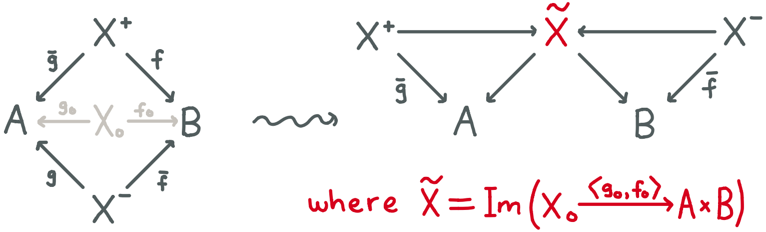
$\mathrm{SymLens}(A,B) \xrightarrow[L]{\text{"pushout"}} \mathrm{SpnLens}(A,B)$



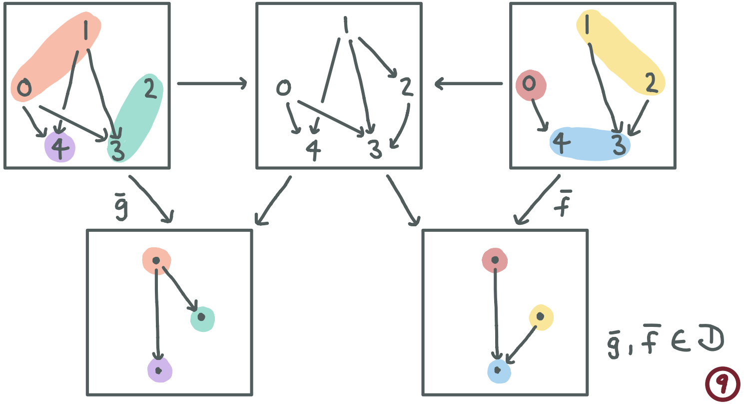
where $\hat{X} = X^{+} \sqcup_{X_0} X^{-}$



$\mathrm{SymLens}(A,B) \xrightarrow[R]{\text{"factorise"}} \mathrm{SpnLens}(A,B)$



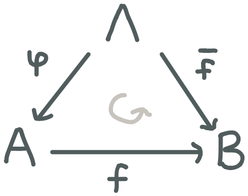
where $\tilde{X} = \mathrm{Im}\left(X_0 \xrightarrow{\langle g_0, f_0 \rangle} A \times B\right)$



Summary

GOAL 1: Develop a diagrammatic framework for lenses.

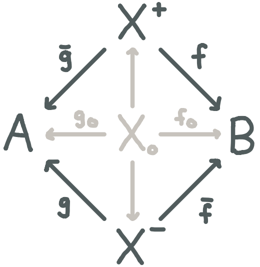
RESULT:



$\varphi \in \mathcal{E}$

$\bar{f} \in \mathcal{D}$

Asymmetric lens

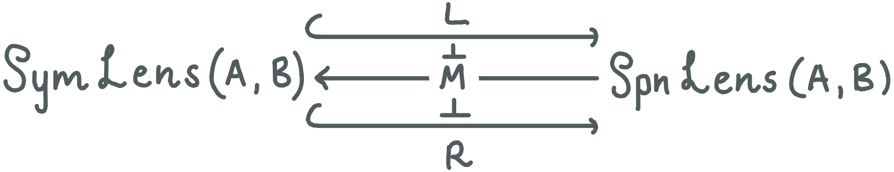


$\bar{g}, \bar{f} \in \mathcal{D}$

Symmetric lens

GOAL 2: Understand the relationship between symmetric & asymmetric lenses.

RESULT:



where R is reflective & L is coreflective