

Question	Asker Name	Answer
Why not use the zoom chat instead of the Q&A?	Viliam Vadocz	I think Q&A is preferable. Allows for upvotes
If we are not allowed to peek into the objects, how do we define useful morphisms?	Karthikeyan Natesan Ramamurthy	live answered
If we are not allowed to peek into the objects, how do we define useful morphisms?	Karthikeyan Natesan Ramamurthy	Of course we first have to be aware of what our objects are to define meaningful morphisms (and that is the crux when defining your category). Once you have that data settled down it is the morphisms that allow you to have the bird's eye view on your global (and also local) structure.
If we are not allowed to peek into the objects, how do we define useful morphisms?	Karthikeyan Natesan Ramamurthy	I think this is rooted in the https://ncatlab.org/nlab/show/Yoneda+lemma
If we are not allowed to peek into the objects, how do we define useful morphisms?	Karthikeyan Natesan Ramamurthy	'@Yivan yes, the spiritual essence of the Yoneda lemma is that, at least from the perspective of category theory, an object has no internal essence, it is fully defined by its relationships with all other objects.

<p>If we are not allowed to peek into the objects, how do we define useful morphisms?</p>	<p>Karthikeyan Natesan Ramamurthy</p>	<p>But unless you define the category you have no Yoneda lemma. :-) So it's not going to help in this case. But yes, one of the gazillion ways of looking at Yoneda is to say that each object is uniquely determined by all its "generalised elements" I mentioned further below.</p>
<p>does $g \circ f$ need to be unique?</p>	<p>ashwath</p>	<p>live answered</p>
<p>If g and f are invertible, is it a category?</p>	<p>jules tsukahara</p>	<p>live answered</p>
<p>If g and f are invertible, is it a category?</p>	<p>jules tsukahara</p>	<p>Categories where every arrow is invertible is called a groupoid. As the name suggests it is a generalisation of a group, and has been around before the concept of a category has been defined.</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>live answered</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>Fun fact: you can construct a category over any graph</p>

<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>'@Filip aka the free category generated by a quiver</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>i guess you need to add all self loops and make all arrow composable?</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>For example, a semigroup lacks an identity and itself can not be seen as a category. However, you can often still define morphisms between two semi-groups, so can form a category of all semigroups.</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>is every category a subcategory of the free category on some quiver?</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>'@Matthew yes, with some asterisks. for example you need to define "sub" to mean a quotient operation where you declare some morphisms to be identical to others, rather than a sub-graph.</p>

<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>this is similar (in fact almost identical!) to how any finite group can be seen as isomorphic to a free group quotiented by some relations</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>Yes, it should be. You can forget the category structure and just look at it as a graph. Then you can construct the "free" category over that graph. This doesn't make your original category a subcategory. But you can perform a suitable "quotient construction" as Tali says.</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be "exhaustive"?</p>	<p>Giacomo Aldegheri</p>	<p>So not as a subcategory but as a 'Quotient' Category by adding relations? Awesome, thanks for the help</p>

<p>Does category theory also deal with non-categories, or is the concept general enough to be “exhaustive”?</p>	<p>Giacomo Aldegheri</p>	<p>'@Jules it's more that the morphisms of the free category represent *paths* of any finite length in the original graph. you can compose paths when they are head to tail. for example, the morphisms between objects a and b in the free category on a graph is the set of paths that begin at a and end at b</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be “exhaustive”?</p>	<p>Giacomo Aldegheri</p>	<p>'@Jules then the "empty path", which immediately starts and ends at a vertex a, is exactly the (unique) identity morphism in the category.</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be “exhaustive”?</p>	<p>Giacomo Aldegheri</p>	<p>'@Jules I don't refer to category theory at all but the way it works is visualized here (https://quivergeometry.net/path-groupoids/) and its the same idea</p>
<p>Does category theory also deal with non-categories, or is the concept general enough to be “exhaustive”?</p>	<p>Giacomo Aldegheri</p>	<p>'@Matthew exactly. you're welcome!</p>
<p>What do you mean by drops a lot of data?</p>	<p>Ibraheem Muhammad Moosa</p>	<p>What do you mean by drops a lot of data?</p>
<p>all are same?</p>	<p>ashwath</p>	

There is a unique one!	Franck Albinet	live answered
Constant fn?	Dobrik Georgiev	live answered
constant?	Victoria Klein	live answered
What does "up to isomorphism" mean in Cat theory? (i.e. what's an Isomorphism)	Dobrik Georgiev	Petar will explain that in a couple of slides
isn't what is in the object makes A different from B? if we ignore what is in the object then everything in a set become the same? like a molecular graph has different atoms because what is inside atoms actually defines what type of atom it is?	Amina Mollaysa	In our context the sets themselves are the objects, what Petar is saying is that we won't talk about the elements of the sets, but the sets themselves differ from each other by things like cardinality
isn't what is in the object makes A different from B? if we ignore what is in the object then everything in a set become the same? like a molecular graph has different atoms because what is inside atoms actually defines what type of atom it is?	Amina Mollaysa	Good point. This is why we need to carefully distinguish between being isomorphic and being equal. Isomers in Chemistry would be isomorphic (in a suitable sense), but not equal.

<p>isn't what is in the object makes A different from B? if we ignore what is in the object then everything in a set become the same? like a molecular graph has different atoms because what is inside atoms actually defines what type of atom it is?</p>	<p>Amina Mollaysa</p>	<p>'@Filip makes me think that a nice illustration of the maxim that "objects are defined by their relationships" for chemistry would be that nuclear isotopes manifest almost no chemical difference (except for slight changes in diffusion and reaction rates due to mass and maybe nuclear spin). so their chemical relationships with other molecules are the same, and we can treat them as the same kind of element "most of the time" because chemistry (=category theory) won't be able to tell the difference between them.</p>
<p>There is B morphisms like that?</p>	<p>yobibyte</p>	<p>live answered</p>
<p>cardinality of B</p>	<p>luchino_prince</p>	<p>live answered</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>you can imagine a category with three objects A, B, C and two morphisms $A \rightarrow B$ and $B \rightarrow C$ and all of the identities, but no morphism $A \rightarrow C$</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>live answered</p>

<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>That's kind of the „wrong“ question as composition is considered to be part of the data.</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>but isn't it the case that you can always easily extend the universe of morphisms to contain the compositions?</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>sure, but then you may be dealing with a different universe. for example, if you are trying to describe some phenomenon using categorical language and you discover that compositionality is not satisfied, you can “add it in yourself,” but you have to acknowledge that the underlying phenomenon was not actually categorical</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>perhaps someone else can weigh in with such a real-world example, i can't think of any at the moment</p>

<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>Yes, you have the right intuition. Given a graph, you can „extend“ this graph and get a category. That construction is called the „free category“ on the given graph.</p> <p>The true problem is that this free category becomes HUGE.</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>regardless of the real world example, i think i see it now more clearly, thanks. (If anyone has a non-trivial example, then I would still appreciate it)</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>'@Piotr. Take as objects the points in space (say a torus = doughnuts), and the arrows to be paths between the points. This does not form a category unless you are very careful how you model continuous paths mathematically.</p>
<p>could you give an example of a non-category that doesn't satisfy the compositionality axiom?</p>	<p>Piotr Piękos</p>	<p>e.g. the graph of lovers, the objects are people, and a connection between A and B means "A feels attraction to B" or "A likes B"</p>

Describes 'inside of B' in a sense?	Shivam	live answered
Don't you still need to have some information about the set? In this case the cardinality?	Andrei Manolache	Just the set morphisms into and out of the object encode the cardinality.
Don't you still need to have some information about the set? In this case the cardinality?	Andrei Manolache	I see, but we still need to know something about the (unique) morphisms, isn't that equivalent to knowing the cardinality of the set?
feels similar to the notion that each point (X, Y) in \mathbb{R}^2 is isomorphic to the vector from the origin to (X, Y)	Max Cembalest	
Might be useful for beginners to know: this concept of the a morphism that tells you about the contents of an object that technically category theory isn't allowed to directly "look inside" is called a "subobject classifier"	Tali Beynon	more precisely: the object for which the morphisms from other objects to it represent sub-objects of those objects, is called a sub-object classifier!

<p>Might be useful for beginners to know: this concept of the a morphism that tells you about the contents of an object that technically category theory isn't allowed to directly "look inside" is called a "subobject classifier"</p>	<p>Tali Beynon</p>	<p>live answered</p>
<p>Might be useful for beginners to know: this concept of the a morphism that tells you about the contents of an object that technically category theory isn't allowed to directly "look inside" is called a "subobject classifier"</p>	<p>Tali Beynon</p>	<p>Subobject classifiers are already a bit too restrictive. I think a better way to think about this is the notion of a "generalised element". So an arrow $f: a \rightarrow b$ would be a generalised element of shape a in b. Would really need to draw some pictures here to make it more visual.</p>
<p>Might be useful for beginners to know: this concept of the a morphism that tells you about the contents of an object that technically category theory isn't allowed to directly "look inside" is called a "subobject classifier"</p>	<p>Tali Beynon</p>	<p>@Filip you're right, thanks! an interesting example of that would be in the category of categories. a generalized element is then a diagram. which makes sense: diagrams are parts of a category as seen through a keyhole.</p>

<p>A graph can be considered an object up to isomorphisms? In that case, graph isomorphisms are the id morphisms on the objects?</p>	<p>Alberto Colombo</p>	<p>There's a category of graphs, whose objects are graphs and morphisms are graph morphisms. The isomorphisms are a subset of morphisms. (Called the core subcategory)</p>
<p>If I understand correctly, the Set category has all the sets as its objects? How do we define the universe of sets, or do we even need to do that?</p>	<p>Oumar Kaba</p>	<p>The universe is not a set, so you do not need to define it in Set :)</p>
<p>If I understand correctly, the Set category has all the sets as its objects? How do we define the universe of sets, or do we even need to do that?</p>	<p>Oumar Kaba</p>	<p>You might want to look at the concepts of "small categories" and "large categories" that are designed to deal with these cardinality issues. I believe the category of sets is a large category.</p>
<p>If I understand correctly, the Set category has all the sets as its objects? How do we define the universe of sets, or do we even need to do that?</p>	<p>Oumar Kaba</p>	<p>Yeah there's some more specific theory to dealing with cardinality issues in the collection of objects of categories, they won't be needed for this introduction (but by all means look it up if you find it interesting!)</p>

<p>If I understand correctly, the Set category has all the sets as its objects? How do we define the universe of sets, or do we even need to do that?</p>	<p>Oumar Kaba</p>	<p>This cannot be resolved. You can call them universes, cumulative hierarchies, classes You will run into these problems eventually. At some point your categories become too big for any form of set theory you want to use to model category theory. The only way out would be an axiomatic theory of categories. Bill Lawvere has developed some ideas in this direction called ETCS: Elementary Theory of the Category of Sets.</p>
<p>Do we consider all single-element sets the same object?</p>	<p>Viliam Vadoz</p>	<p>Not the same object, but isomorphic objects.</p>
<p>Does the category of sets have to include all functions from each set to each other set? (As in, are all morphisms included by definition)</p>	<p>Charles London</p>	<p>yes</p>
<p>Does the category of sets have to include all functions from each set to each other set? (As in, are all morphisms included by definition)</p>	<p>Charles London</p>	<p>live answered</p>

Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	yes, the empty function petar just mentioned.
Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	live answered
Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	$ 0 ^{ 0 }$ = undefined, hence my question
Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	As in some other settings, the convention $0^0=1$ should be used here
Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	0^0 is undefined only from a calculus perspective. from a combinatorics/set theoretic perspective is 1.
Does there exist $\text{Hom}(\emptyset, \emptyset)$ morphism?	Dobrik Georgiev	All objects require an identity morphisms so the homset can't be empty or undefined.
why can't we force $ B $ to be equal to 1? (for $ B ^{ A }=1$)	ashwath	It's for all B
why can't we force $ B $ to be equal to 1? (for $ B ^{ A }=1$)	ashwath	For A to be an initial object, it needs to hold for all B
so the morphisms in the opposite categories don't have to be inverses of the original morphisms? Axioms don't assume that	Piotr Piękos	live answered

What's the meaning of opposite category of set category? ie what do the morphisms mean?	Ibraheem Muhammad Moosa	What's the meaning of opposite category of set category? ie what do the morphisms mean?
What's the meaning of opposite category of set category? ie what do the morphisms mean?	Ibraheem Muhammad Moosa	live answered
What's the meaning of opposite category of set category? ie what do the morphisms mean?	Ibraheem Muhammad Moosa	it is isomorphic to a boolean algebra category, actually: https://math.stackexchange.com/questions/980933/what-is-the-opposite-category-of-set
What's the meaning of opposite category of set category? ie what do the morphisms mean?	Ibraheem Muhammad Moosa	It's a purely formal construction. It might not have any immediate underlying meaning other than there exists a morphism $A \rightarrow B$ whenever there exists a morphism $B \rightarrow A$ in the original category.
What's the meaning of opposite category of set category? ie what do the morphisms mean?	Ibraheem Muhammad Moosa	Dual / opposite categories are a syntactic construction. But once you can establish an isomorphism (in fact equivalence is often sufficient) with a concrete category, then you have what we like to call a duality.

<p>Is there any intuitive way to think about the opposite of a category we already have familiarity with?</p>	<p>Matthew Pugh</p>	<p>probably in the language of sets would be useful for me :)</p>
<p>Is there any intuitive way to think about the opposite of a category we already have familiarity with?</p>	<p>Matthew Pugh</p>	<p>Once Petar introduced functors, there's a nice example in the pre-image set. This gives a functor $\text{Set}^{\text{op}} \rightarrow \text{Set}$, mapping set X to the powerset PX, and a function $f: X \rightarrow Y$ to a function $PY \rightarrow PX$, mapping a subset $U \subseteq Y$ to its pre-image $f^{-1}(U)$. To flip the direction, we need the opposite category.</p>

<p>Is there any intuitive way to think about the opposite of a category we already have familiarity with?</p>	<p>Matthew Pugh</p>	<p>Maybe not the most intuitive way of thinking about opposite categories, but it is at the heart of what categorical duality is: Think of a formal definition of the theory of a category; i.e. the language and a basic set of syntactic rules, like you would define a programming language. If you now do the following operation on this formal theory: switch domain and codomain of each arrow. You end up with the same theory. On your model side this means, that every model of the theory, i.e. a concrete category (a particular implementation of your programming language) comes with a sister model, the opposite category. This is the mother of all dualities in mathematics and beyond.</p>
<p>Is there any intuitive way to think about the opposite of a category we already have familiarity with?</p>	<p>Matthew Pugh</p>	<p>'@Filip well said! it really is kind of an instance of metamathematical gauge fixing</p>
<p>you get a surjective $A \rightarrow C$?</p>	<p>Max Cembalest</p>	
<p>composition is a function</p>	<p>Piotr Piękos</p>	
<p>You will have $A \rightarrow C$ defined</p>	<p>yobibyte</p>	
<p>All elements of C can be mapped from A?</p>	<p>Milena Djordjevic</p>	
<p>fg is surjective if f and g are surjective</p>	<p>Matthew Pugh</p>	

'@Filip isn't that construction a non-category when you choose shortest paths, however when you look at the picture upto homotopy invariance on a surface without holes that becomes a category as Piotr mentioned	Abdullah Canbolat	Even if you would choose paths as continuous maps $[0,1] \rightarrow \text{torus}$ (not necessarily shortest), then you would not get an associative composition. Yes, you can pass to homotopy classes, but you can also be more clever with your parameterisation to make this work.
You get transitivity?	Fredi Mino	
codomain of the composition would be same as g	Nilay	
It will follow associative rule	Zarreen	
Clarification: Sets in Set cat. do not need to be of the same type?	Dobrik Georgiev	i.e. we can have sets of numbers, pairs, etc.
Clarification: Sets in Set cat. do not need to be of the same type?	Dobrik Georgiev	In the Set cat, there's no typing of the sets.
Clarification: Sets in Set cat. do not need to be of the same type?	Dobrik Georgiev	Well, there kind of is, because each set is its own type. (See categorical logic/Mitchel-Benabou language)
Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is \mathcal{C}^{op} no longer Set?	Charles London	As in, it is no longer required that morphisms are functions?

<p>Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is C^{op} no longer Set?</p>	<p>Charles London</p>	<p>the Op construction is purely formal the arrows in SetOp are just “formally” reversing the arrows from Set, but they do not actually represent going “backwards”, and they do not have to be isomorphisms in fact, in SetOp, /every/ arrow gets turned around, including non-invertible functions</p>
<p>Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is C^{op} no longer Set?</p>	<p>Charles London</p>	<p>Thank you!</p>
<p>Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is C^{op} no longer Set?</p>	<p>Charles London</p>	<p>live answered</p>

<p>Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is C^{op} no longer Set?</p>	<p>Charles London</p>	<p>Yeah, as Daniel says you can see it as just choosing the opposite convention for what an arrowhead means. Kind of like how with groups you can choose group multiplication to be left-to-right or right-to-left, switching the convention can be compensated by moving to the "opposite group". it's a kind of discrete gauge choice if you are inclined to take a physics perspective.</p>
<p>Is it always possible to define an opposite category? At least in Set it seems that it is only possible if all the functions are isomorphic? Or is C^{op} no longer Set?</p>	<p>Charles London</p>	<p>At its heart categorical duality is due to the self-duality of the theory of a category. I have elaborated that in a bit more depth further above.</p>
<p>projections are surjections right?</p>	<p>Ibraheem Muhammad Moosa</p>	<p>projections are surjections right?</p>
<p>projections are surjections right?</p>	<p>Ibraheem Muhammad Moosa</p>	<p>yes</p>
<p>projections are surjections right?</p>	<p>Ibraheem Muhammad Moosa</p>	<p>live answered</p>
<p>projections are surjections right?</p>	<p>Ibraheem Muhammad Moosa</p>	<p>In the category of sets, yes.</p>
<p>Is it sensible to think of the Cartesian product as, say, cartesian coordinates with some minimal number of dimensions?</p>	<p>Ieva</p>	<p>(as an example)</p>

Is it sensible to think of the Cartesian product as, say, cartesian coordinates with some minimal number of dimensions?	leva	In that X would then be something of higher dimensionality containing both dimensions A and B
Is it sensible to think of the Cartesian product as, say, cartesian coordinates with some minimal number of dimensions?	leva	I think that's sensible.
Are there any cases where we want to define "isomorphism between morphisms"?	Yivan Zhang	Yes, if you want to study 2-category theory.
Are there any cases where we want to define "isomorphism between morphisms"?	Yivan Zhang	That'd be in a bicategory / 2-category, which is out of the scope of this course so far. Key example: homotopies can be isomorphisms between continuous functions.
Are there any cases where we want to define "isomorphism between morphisms"?	Yivan Zhang	Thanks! I only know the names, but I haven't encountered a problem in machine learning where we need to go that far yet, hence the question.
So if I understand correctly, $A \times B$ is only defined up to isomorphism?	Oumar Kaba	live answered

<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>That's one valid choice of topology for products in Top.</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>And is also the cartesian product in the category of topological spaces and continuous maps as arrows.</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>however box topology does not preserve topological properties for arbitrary products, do we need another definition for arbitrary product category or is this definition enough to cover these</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>Sorry, I am not quite sure what you are referring to here. What matters is the universal property. Maybe we mean different things by box topology. I have taken that to mean product topology.</p>

<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>umm no, box topology refers to the topology where products of open sets are in the topology, but the product topology is the weakest topology that the projection mappings are continuous. for infinite products of topologies therefore product topology is weaker than box topology and for example compactness on box topology is not preserved</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>what i was asking about this infinite product of topological objects.</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>Okay, in that case no; the infinite product will carry the initial topology w.r.t the projections, not the box topology.</p>
<p>after Petar said $A \times B$ is minimal, i wonder if box topology can be considered as a product object in the category of point set topology</p>	<p>Abdullah Canbolat</p>	<p>ok, thank you :)</p>

Is the coproduct a set of sets?	Siavash Sakhavi	It's the disjoint union of sets, which is not the same.
Is the coproduct a set of sets?	Siavash Sakhavi	Noted. Thank you
union	Jeffrey Nickerson	Close, but it's a disjoint union
Are there objects that are self-duals in the set category?	Oumar Kaba	When taking the opposite of a category we're inverting the morphisms but keeping the objects the same
Are there objects that are self-duals in the set category?	Oumar Kaba	OK makes sense, then to reformulate, are there objects such that the morphisms are preserved when taking the dual?
Are there objects that are self-duals in the set category?	Oumar Kaba	Endmorphisms, yes, as they have the same domain and codomain.
all possible combinations of elements a set which is combination of A and B?	Amina Mollaysa	

<p>Category theory is called often as a general math language, what means we could use it to express any math concept without additional words. Can we express word "any" in product definition in a categorical way? What we can use - natural transformation?</p>	<p>Stanislav Kapulkin</p>	<p>The universal quantifier appears as the right adjoint to the pullback. It's not so easy to elaborate here but feel free to ask for more details on Zulip!</p>
<p>Category theory is called often as a general math language, what means we could use it to express any math concept without additional words. Can we express word "any" in product definition in a categorical way? What we can use - natural transformation?</p>	<p>Stanislav Kapulkin</p>	<p>In general we have what we call Categorical Logic, where we can interpret theories in (fractions of) first order logic. How much we can interpret depends on how much stuff we can do in said category.</p>
<p>Can the product be defined as an object with two diagrams, with $A \rightarrow A \times B \rightarrow A$ for inclusion, projection and identity, and also for B?</p>	<p>Shivam</p>	<p>In general, cartesian categories don't have an inclusion. This is true for bicartesian categories, in which the product and coproduct are the same. Example: vector spaces.</p>
<p>isn't B^A the same as $\text{Hom}(A, B)$?</p>	<p>jules tsukahara</p>	<p>B^A lives in the category, $\text{Hom}(A, B)$ is a set.</p>

isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	in the category of sets, yes.
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	live answered
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	but, in the general case, B^A is an object, not a set.
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	so not a subset of $\text{Mor}(C)$.
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	it doesn't even need to be a set in a more weird category.
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	ok i'll need to check the definition in details later, thanks
isn't B^A the same as $\text{Hom}(A,B)$?	jules tsukahara	The idea of exponentials is that they are objects representing the set of all morphisms *internally* to the category as one of its objects.
Is it the case that B^A is some sort of a terminal object?	Dobrik Georgiev	No, that'd require that for any X there's a unique map $X \rightarrow B^A$, but by currying, any map $X \times A \rightarrow B$ gives such a map.
How are $\text{Hom}(A,B)$ and the exponential B^A related?	Federico	Yes: $\text{Hom}(A, B)$ is isomorphic as a set to $\text{Hom}(1, B^A)$
How are $\text{Hom}(A,B)$ and the exponential B^A related?	Federico	number of elements in the set. so $ \text{Hom}(A, B) = B ^{ A }$. But please double check
currying	Brian Lee	
currying?	Daniel Gonzalez Cedre	
currying	Viliam Vadocz	
currying	Jeffrey Nickerson	
Empty?	Franck Albinet	
empty and power?	volodymyr ky	

<p>Why was the case \emptyset was terminal in Rel?</p>	<p>Dobrik Georgiev</p>	<p>live answered</p>
<p>In the set category, why don't we define a morphism going from any set to the empty set?</p>	<p>Oumar Kaba</p>	<p>set functions must have outputs defined for their inputs, and there must be an output defined for /every/ input</p> <p>if you take a nonempty domain A and try to define a function with codomain \emptyset, you run into a problem because you can't define outputs for any of the inputs from A</p>
<p>In the set category, why don't we define a morphism going from any set to the empty set?</p>	<p>Oumar Kaba</p>	<p>live answered</p>
<p>In the set category, why don't we define a morphism going from any set to the empty set?</p>	<p>Oumar Kaba</p>	<p>A follow up, to the above question. Was it 'our' decision to define morphisms as fns in Set category? Or are other categories on sets, with different morphisms definition, that still satisfy axioms of category?</p>

<p>In the set category, why don't we define a morphism going from any set to the empty set?</p>	<p>Oumar Kaba</p>	<p>i think the natural categorization of set theory is the one that makes the functions into morphisms precisely because, in other areas of math, the transformations between objects are formalized as set functions with additional properties or restrictions</p> <p>in this way, the Set category as defined gives a good introduction to category theory because it has many of the interesting objects and constructions that show up in other areas</p> <p>i'm sure you can take other definitions for morphisms in Set, but they would be harder to interpret without already being familiar with some cat theory (i would imagine)</p>
<p>In the set category, why don't we define a morphism going from any set to the empty set?</p>	<p>Oumar Kaba</p>	<p>You could for example use relations between two sets as morphisms: so objects are sets and arrows are relations. Composition is composition of relations.</p>
<p>there's only one object</p>	<p>Ieva</p>	
<p>There is one object</p>	<p>luchino_prince</p>	
<p>one object</p>	<p>Flaviu Iepure</p>	
<p>one element</p>	<p>Walker# Ian A</p>	
<p>the id</p>	<p>Flaviu Iepure</p>	

anyone see a possible application of groupoids into geometric deep learning?	caio	anyone see a possible application of groupoids into geometric deep learning?
anyone see a possible application of groupoids into geometric deep learning?	caio	live answered
more objects, under some conditions, would give us a groupoid	luchino_prince	
slightly tangential question: is there a process in CT which relates things like the category induced by a single group BG to the category of all groups Grp?	jules tsukahara	I might look to Lawvere theories, operads, and similar things.
slightly tangential question: is there a process in CT which relates things like the category induced by a single group BG to the category of all groups Grp?	jules tsukahara	I guess you could think of the functor $\text{Groups} \rightarrow \text{Cats}$, mapping a group as an object G in the category of groups, to the category BG , which is an object in the category of categories.
What about defining categories for groups using elements of a set as objects and a group action for morphisms?	Oumar Kaba	That also works and is called the action groupoid of the group acting on itself.

What about about defining categories for groups using elements of a set as objects and a group action for morphisms?	Oumar Kaba	Interesting, thank you!
What about about defining categories for groups using elements of a set as objects and a group action for morphisms?	Oumar Kaba	'@Oumar from a graph theory perspective, a graph that generates that category you just defined is called a Cayley graph. generalizing it to actions on other sets gives interesing edge-colored graphs: https://quivergeometry.net/action-groupoids/
Can we get opposite vategory with a functor?	Ibraheem Muhammad Moosa	Can we get opposite vategory with a functor?
Can we get opposite vategory with a functor?	Ibraheem Muhammad Moosa	When there's a functor $C \rightarrow C^{\text{op}}$, this functor is called the dagger and the category is called a dagger category. Key example: vector spaces with the (conjugate) transpose.
Can we get opposite vategory with a functor?	Ibraheem Muhammad Moosa	You can consider any Functor $F: C \rightarrow D$ also as a functor $F: C^{\text{op}} \rightarrow D^{\text{op}}$
kinda like a group homomorphism where the operation in both groups is composition?	senri	kinda like a group homomorphism where the operation in both groups is composition?

kinda like a group homomorphism where the operation in both groups is composition?	senri	Yes, great analogy!
kinda like a group homomorphism where the operation in both groups is composition?	senri	Exactly: functors between categories $BG \rightarrow BH$ for groups B and H are exactly group homoms
is the Functor surjective? or not necessarily	Siavash Sakhavi	Not necessarily, see https://math.stackexchange.com/questions/3288868/is-a-full-functor-not-necessarily-surjective-in-terms-of-objects
is the Functor surjective? or not necessarily	Siavash Sakhavi	Noted. Thank you
Can the necessary functor conditions be described as saying that the functor must always map to a valid category?	Lucas	We also need that this map preserves the identities and composition. Just saying that there is a map between morphisms of the right type is not sufficient.
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	Could you rephrase your question?
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	Is the product of two objects in a category just a definition?
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	yes, this is a construction that is defined by the diagram given earlier in the lecture, but it does not always exist in every category
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	Noted. Thank you.

How is $A \times B$ defined as a morphism?	Siavash Sakhavi	to be clear: this is the definition of an /object/ in the category
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	Yes, and no. You can either think of it as a *property* of a category, the fact that you can form products. You can also define it as a part of structure, i.e. mapping any two objects to a particular choice of their cartesian product.
How is $A \times B$ defined as a morphism?	Siavash Sakhavi	That mapping is actually an example of a bifunctor
Just wanted to say that this is the clearest explanation I've ever had of these concepts I've ever come across	leva	Thank you!
Just wanted to say that this is the clearest explanation I've ever had of these concepts I've ever come across	leva	concurring with this; this is a great and concise lecture
Just wanted to say that this is the clearest explanation I've ever had of these concepts I've ever come across	leva	Thanks! I'll make sure Petar gets this message.
Will you very briefly say what message passing is?	Martha Lewis	live answered

<p>It make sense to consider an “inverse” of a functor?</p>	<p>Lorenzo Giusti</p>	<p>Yes, although it's more common to see the more general idea of "adjoints".</p>
<p>It make sense to consider an “inverse” of a functor?</p>	<p>Lorenzo Giusti</p>	<p>Absolutely, a loose but very important notion thereof are called adjunctions, which are out of scope for this course.</p>
<p>It make sense to consider an “inverse” of a functor?</p>	<p>Lorenzo Giusti</p>	<p>At the functor level you do have a notion of isomorphism (think of the category of (small) categories as objects and functors as arrows) or that of an equivalence, or that of adjunctions as pointed out above.</p>
<p>It make sense to consider an “inverse” of a functor?</p>	<p>Lorenzo Giusti</p>	<p>Thanks! This provides a very concrete sense on how to dive into the abstract fields of math</p>
<p>In the example, it seems like all the possible functors from S_3 to V have exactly the same "structure". Is there a way to call them isomorphic?</p>	<p>Oumar Kaba</p>	<p>They're not all isomorphic; for example, there is a functor mapping S_3 to the trivial vector space. But there is a structure theorem; every such functor decomposes into irreducible representations and there are only 3 of them.</p>

<p>In the example, it seems like all the possible functors from S_3 to V have exactly the same "structure". Is there a way to call them isomorphic?</p>	<p>Oumar Kaba</p>	<p>For a given group G, there is a category of (linear) representations of G, $\text{Rep}G$, whose objects are representations and morphisms are linear equivariant maps. This gives a notion of isomorphism between representations. But as Andrew says, these examples are not iso.</p>
<p>In the example, it seems like all the possible functors from S_3 to V have exactly the same "structure". Is there a way to call them isomorphic?</p>	<p>Oumar Kaba</p>	<p>Thanks!</p>
<p>https://arxiv.org/abs/2203.15544</p>	<p>R Jhirad</p>	
<p>Cylinder</p>	<p>Shivam</p>	
<p>functors??</p>	<p>Flaviu Iepure</p>	
<p>cylinder?</p>	<p>Matthew Pugh</p>	
<p>cross product of circle and line somewhat?</p>	<p>Karthikeyan Natesan Ramamurthy</p>	
<p>but then if you go from this cylinder to the line you may end up same effect as just ignoring the circle and mapping the line, no?</p>	<p>Amina Mollaysa</p>	<p>You might, but see the Fourier transform for an example where you get something interesting :)</p>

<p>This could be something very close to a concept called Fourier-Mukai transform</p>	<p>Federico</p>	<p>We're still working out the theoretical connections but we certainly believe there is considerable shared DNA here :)</p>
<p>Why do the sets need to be finite?</p>	<p>Giacomo Aldegheri</p>	<p>live answered</p>
<p>Within the category of Set is it meaningful to say the set of points of a circle. Isn't the structure which makes a circle a circle forgotten within Set?</p>	<p>Matthew Pugh</p>	<p>If you want to take the illustration literally, you're right that it should take place in the category of topological spaces instead. (the actual paper uses finite sets with no topology)</p>
<p>Within the category of Set is it meaningful to say the set of points of a circle. Isn't the structure which makes a circle a circle forgotten within Set?</p>	<p>Matthew Pugh</p>	<p>That makes sense, thankyou</p>
<p>Why product, not coproduct? (in this cylinder example)</p>	<p>Yivan Zhang</p>	<p>We're looking for a way to "mix" the signals, which the coproduct doesn't accomplish.</p>
<p>How does a generic set W give the shape of a tensor?</p>	<p>Martha Lewis</p>	<p>Technically a tensor shape has a little more structure than just being a finite set, but you can think of a square tensor shape as a product of two finite sets, for example.</p>

Could you go over what exactly W, X, Y and Z represent in that diagram again?	Euan Ong	live answered
Ok, thanks!	Martha Lewis	
What does + mean in $V + V^2$?	Ricardo Carnieri	live answered
Sorry, what paper is Petar talking about?	jules tsukahara	GNN are Dynamic Programmers - NeurIPS'22
Sorry, what paper is Petar talking about?	jules tsukahara	https://arxiv.org/abs/2203.15544
Sorry, what paper is Petar talking about?	jules tsukahara	Not this one, the one on Expressive GNN and reasoning on triplets
How can we use functors to learn symmetries between certain categories?	Nicolás	See eg Bruno's paper https://arxiv.org/abs/2009.06837
Is there a way to 'fix message passing' without making the complexity $O(n^3)$? Or provably that's the minimum.	Dobrik Georgiev	live answered
who was the author mentioned in the expressive graph neural networks comment?	Tali Beynon	live answered
Could the improvement comes from more parameters of V^3 ?	Phúc Lê	live answered
Could the improvement comes from more parameters of V^3 ?	Phúc Lê	live answered

<p>Couldn't you argue that computing two different representations for edges just amounts to doubling the number of channels and disallowing interactions between the two subsets of channels?</p>	Oumar Kaba	live answered
<p>Can you recommend more basic sources about your last topic (message passing)?</p>	Sebastian Thomas (dida)	live answered
<p>Can you recommend more basic sources about your last topic (message passing)?</p>	Sebastian Thomas (dida)	Thanks!
<p>Very nice lecture. Thanks Petar.</p>	Karthikeyan Natesan Ramamurthy	
<p>Thank you!</p>	Filip Bar	
<p>thanks for the great lecture :)</p>	Julius Gruber	
<p>Thank you, Petar!</p>	Adel Ardalan	
<p>Thanks for an amazing lecture! A small request: i think Zoom spits out the content of the Q&A as a text file to the host. Could it be preserved and provided on Zulip later?</p>	Tali Beynon	
<p>Thank you</p>	Flaviu Iepure	
<p>Can you tell us a bit more about the CLRS algorithmic reasoning benchmark if time permits? I didn't quite catch what it was.</p>	jules tsukahara	live answered

thanks for the lecture! it was fantastic	Daniel Gonzalez Cedre	
thank you, it was a great lecture	Piotr Piękos	
On what type of task do you expect this extension of Geometric Deep Learning to be used? I don't know I was able to be clear, seems to me that any task related to euclidian spaces has groups as their transformations	luchino_prince	
Thanks for the great lecture by the way!	jules tsukahara	
Thank you for the great lecture!	Oumar Kaba	
Thank you, Petar!	Jorge Hernandez	
what is the coexponential object in set?	Matthew Pugh	live answered
Thank you, great lecture 👍	Nikita Iserson	
Thanks! Super clear lecture :)	Martha Lewis	
Fantastic lecture - thank you	HAMZA GIAFFAR	
Thanks for the lecture!	Mihael Kovač	
is there a categorical way to think about random walks in GNNs?	Jeffrey Nickerson	live answered

<p>is there a categorical way to think about random walks in GNNs?</p>	<p>Jeffrey Nickerson</p>	<p>Weirdly I just bought this book: https://www.cambridge.org/core/books/random-walks-and-heat-kernels-on-graphs/5B375D343025BCE91C682D49CDD3A1A</p>
<p>Is there any associated code/companion notebooks to the Graph Neural Networks are Dynamic Programmers paper? Or adjacent question, any toy examples (notebooks) we could use to play around and ingest these concepts of message passing? Or we could work on collaboratively as a lecture series outcome?</p>	<p>Franck Albinet</p>	<p>live answered</p>
<p>Is there a sense of concrete set of advantages of networks that pass categorical type checking?</p>	<p>Karthikeyan Natesan Ramamurthy</p>	
<p>where in Zulip should we discuss the exercises?</p>	<p>Samuel Gélineau</p>	<p>In the topic of this lecture</p>
<p>How applicable is CT for learning from high-dimensional data, i.e text, images?</p>	<p>Phúc Lê</p>	<p>live answered</p>

is there a link between what you presented here and Haggai Maron's characterization of linear equivariant maps in graphs?	Oumar Kaba	
Has anyone looked at the neural assembly calculus via the framework of category theory? Reference: https://www.pnas.org/doi/10.1073/pnas.2001893117	Alisa Leshchenko	That looks very interesting!
Has anyone looked at the neural assembly calculus via the framework of category theory? Reference: https://www.pnas.org/doi/10.1073/pnas.2001893117	Alisa Leshchenko	live answered
Are there any works that have looked into either disentanglement of representations in DNNs from a categorical theory perspective?	Aishwarya Balwani	
Where will the slide be available?	Ibraheem Muhammad Moosa	Where will the slide be available?
Thank you!	R Jhirad	
thank you!	senri	thank you!