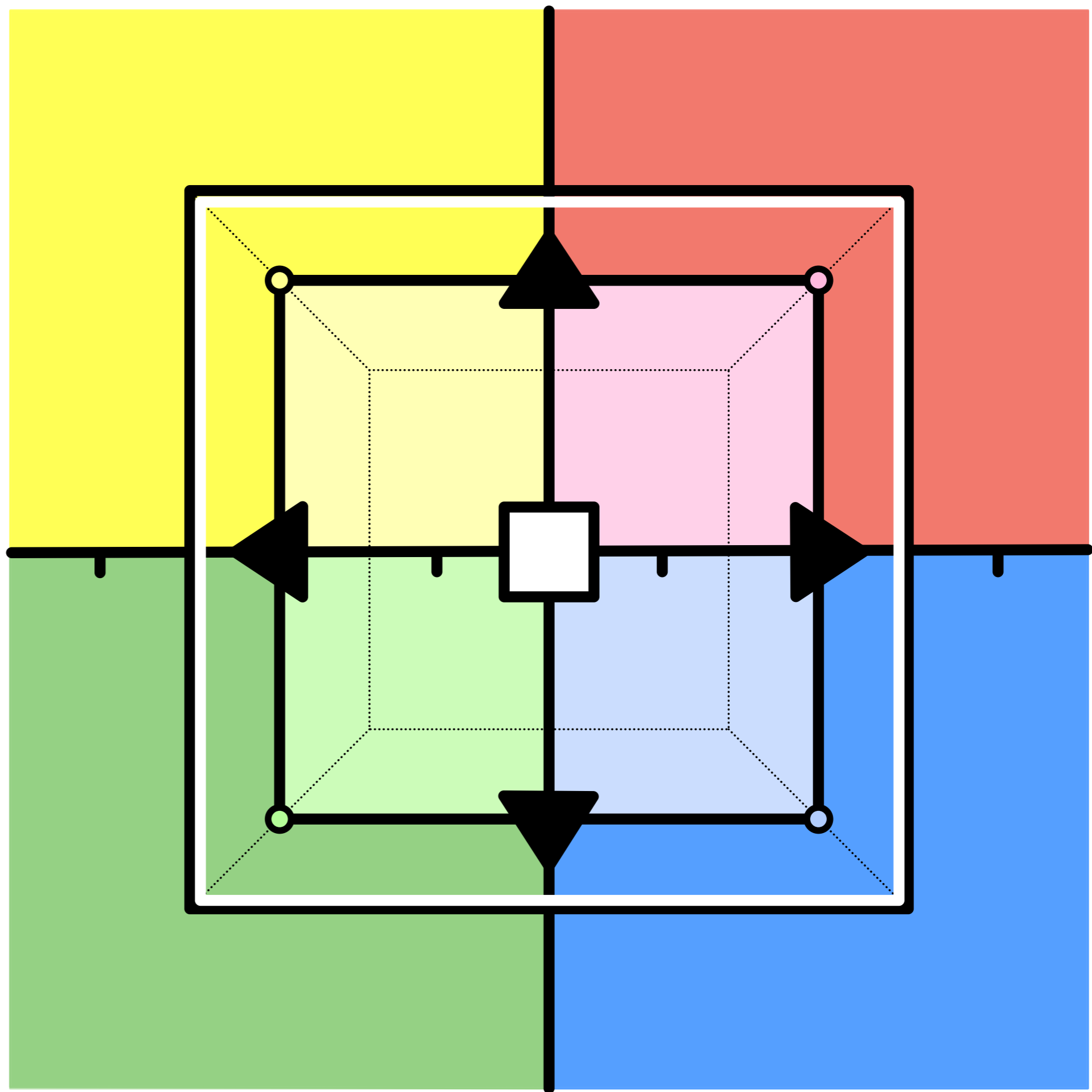


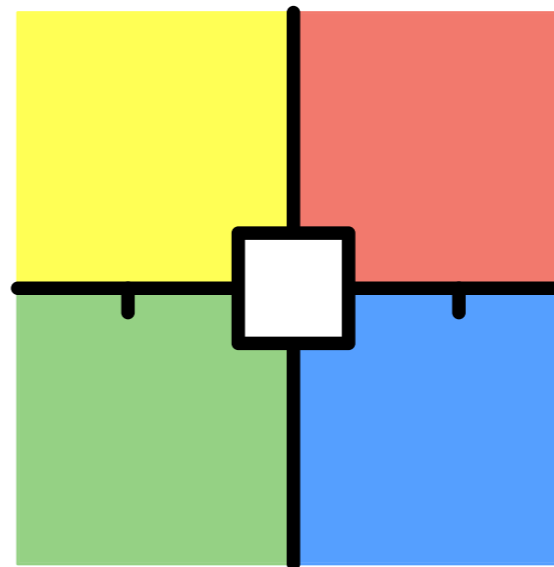
Thinking  
in 3d

Christian  
Williams



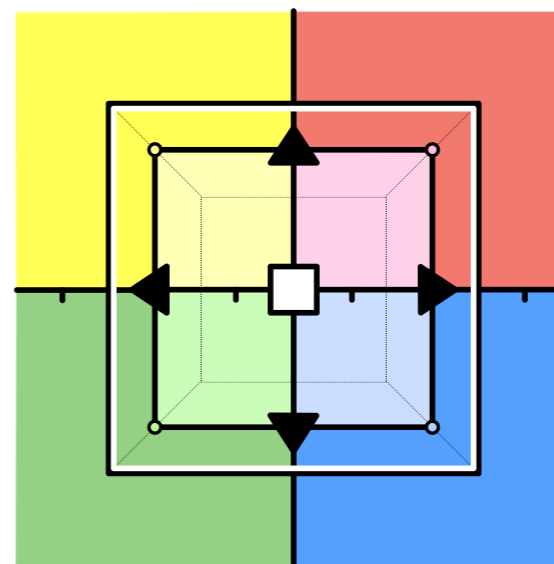
1 Category Theory is Logic

\* a CT  
= an FDC  
= a logic

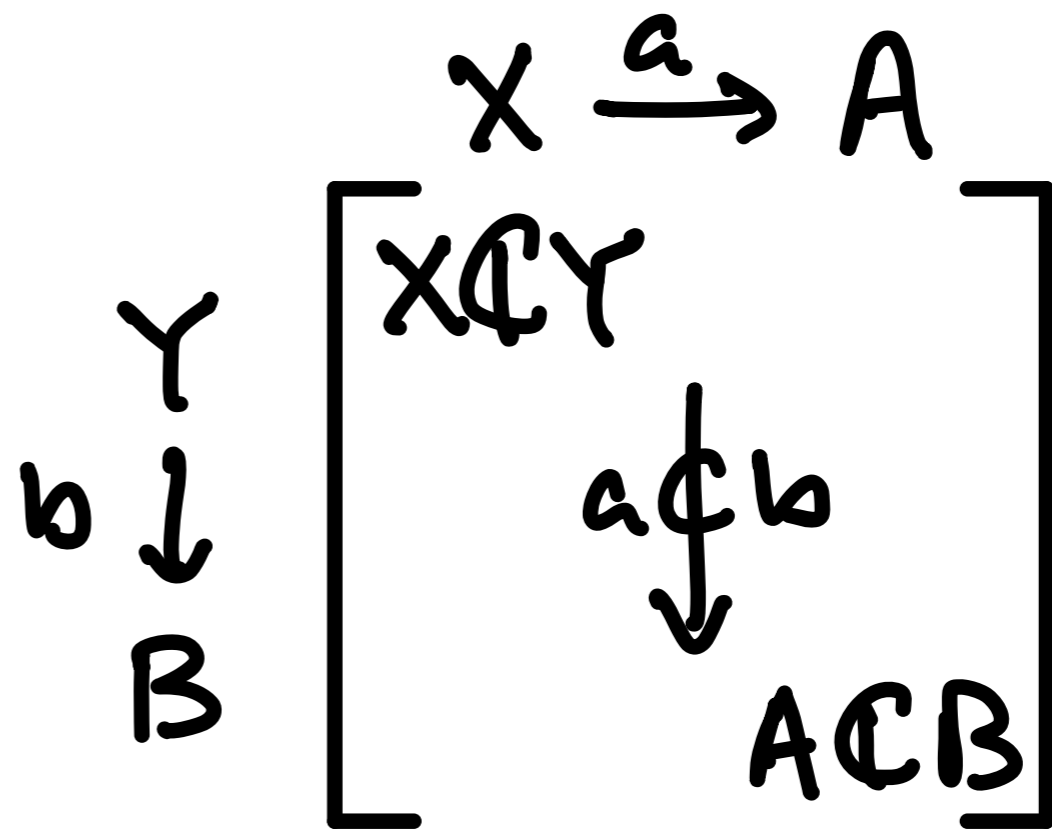
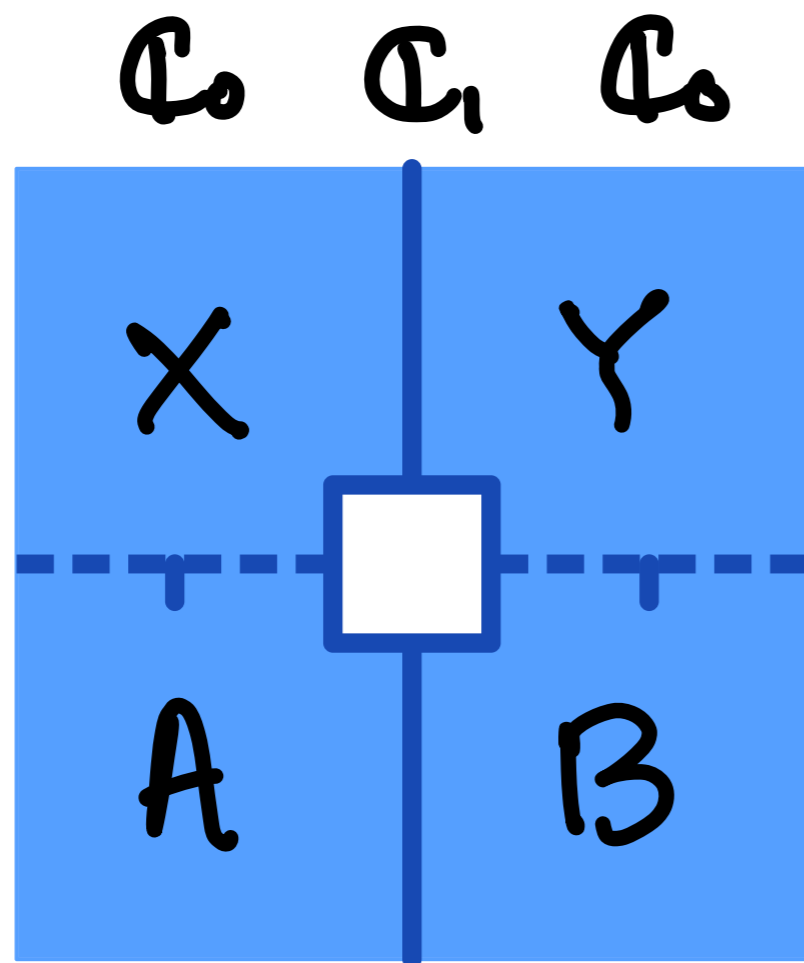


2 Logic is 3-dimensional.

\* FDC is a vibrant triple category

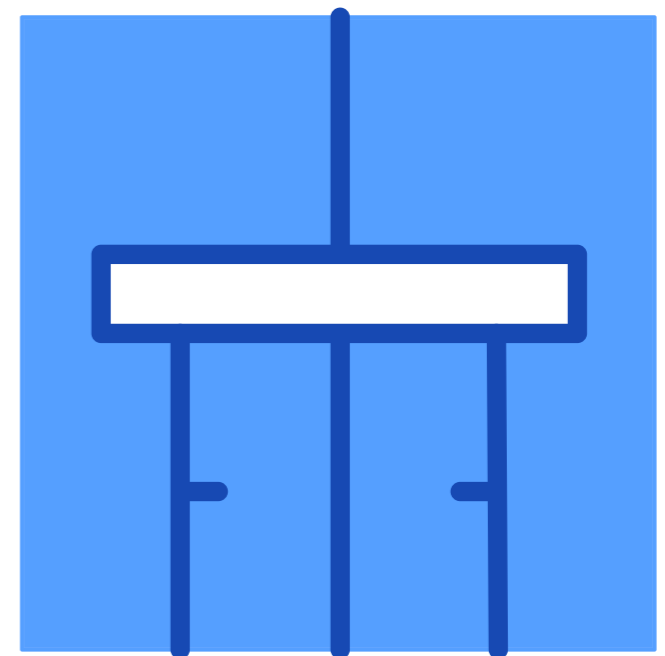


What is a fibrant double category?



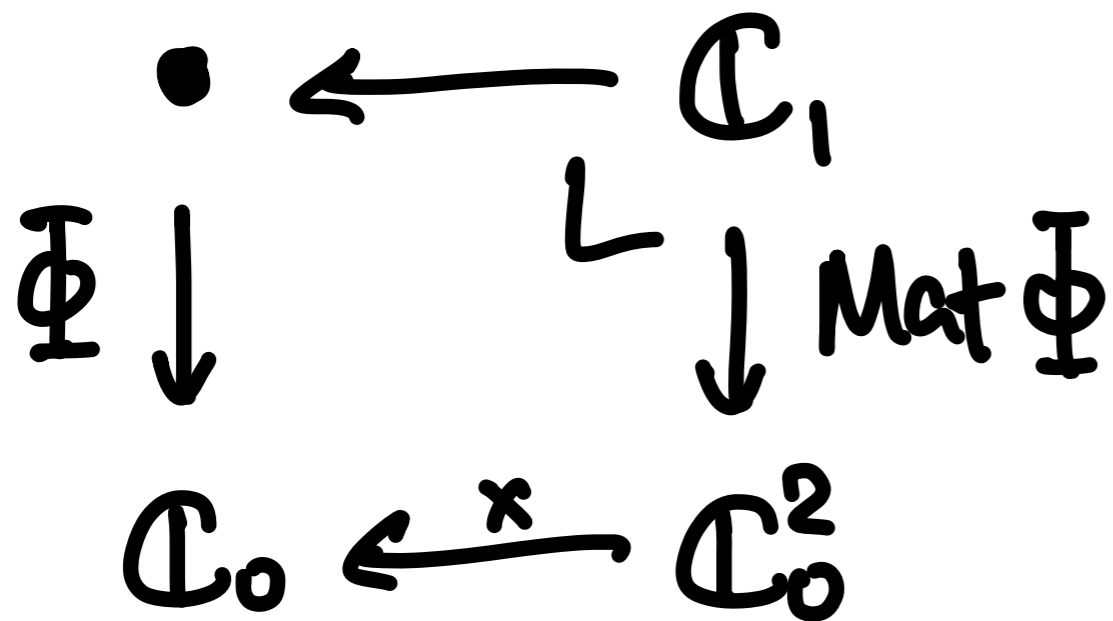
$$\mathbb{C}(a, b)(Q, R) \cong \mathbb{C}_1(Q, aRb)$$

essential to a complete language

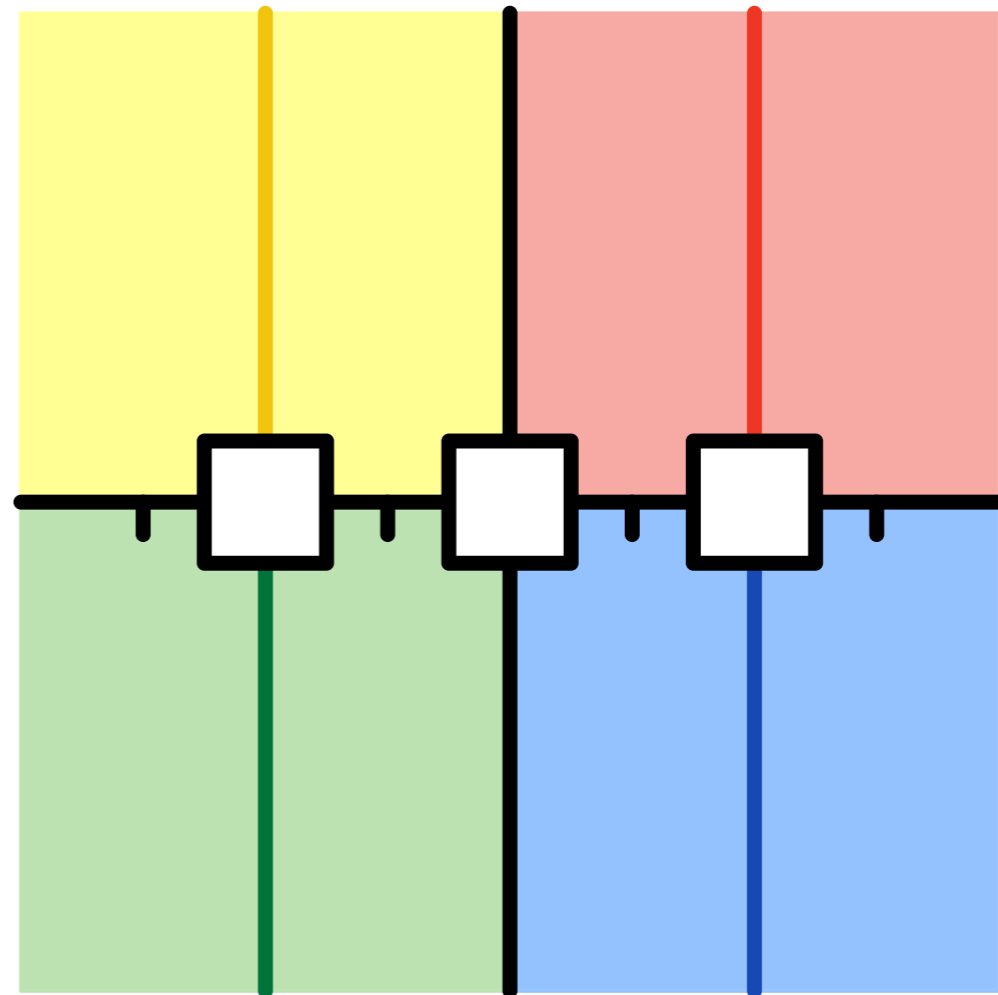


# How do we make FDCs?

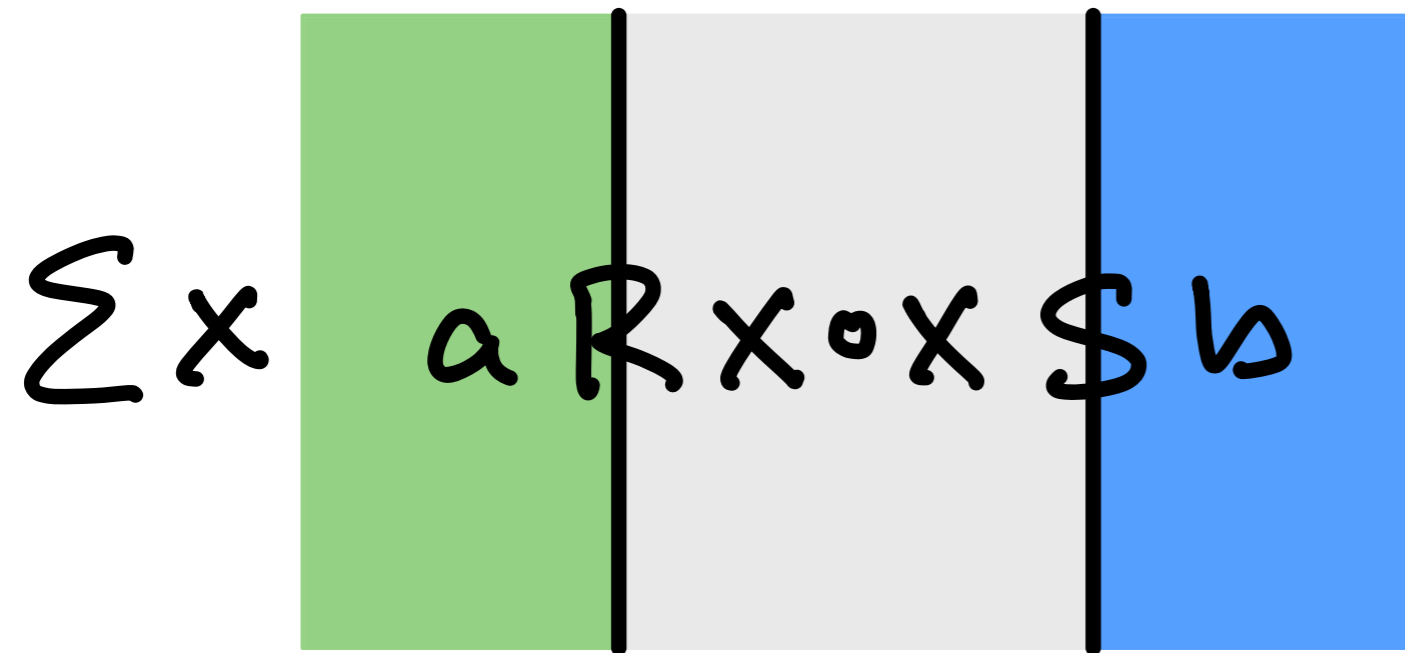
①  $\text{Mat}(-)$   
of a monoidal fibration



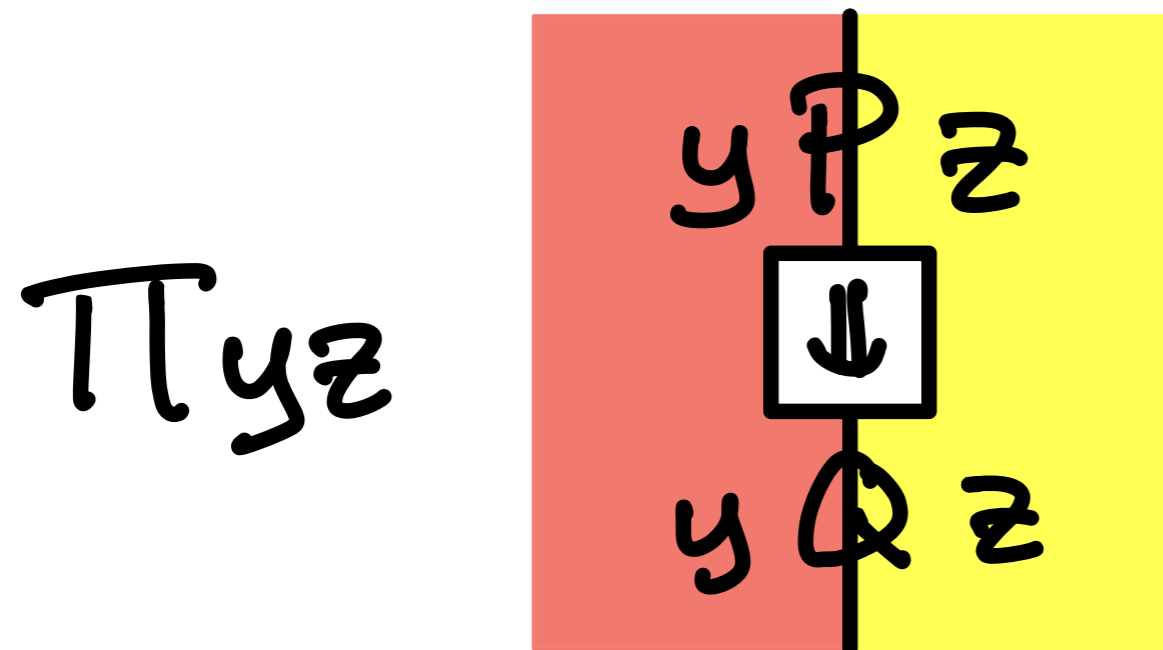
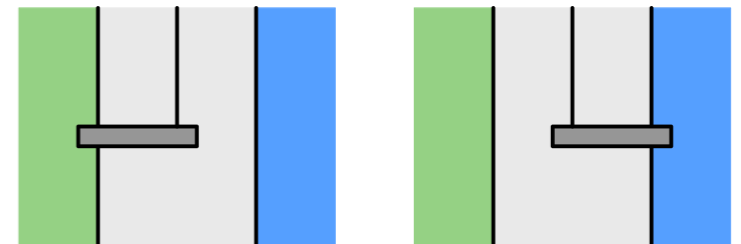
②  $\text{Mnd}(-)$   
of an FDC



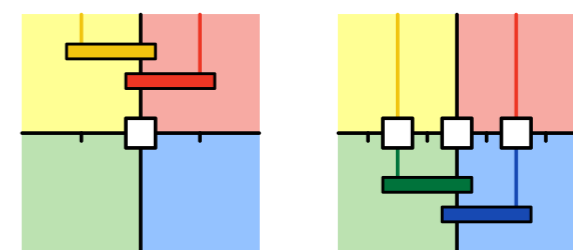
# The Language



judgements  
compose by coend  
("bilinear"  $\exists$ )



inferences  
abstract by end  
("natural"  $\forall$ )



# The Construction

$$\text{Mat}: \text{Fib} \rightarrow \text{Dbl}$$

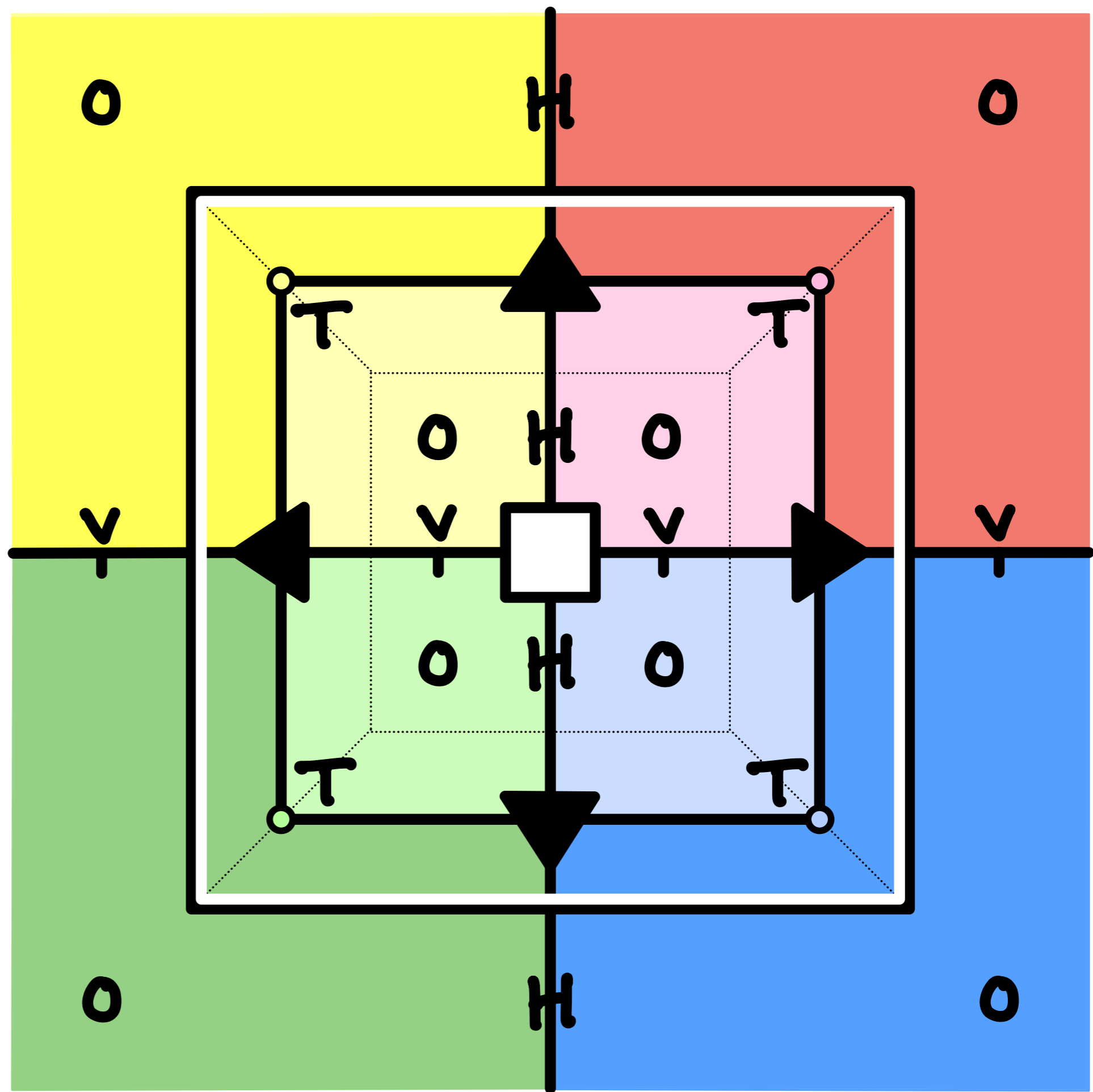
$$\text{Cat}(\text{Mat}): \text{Cat}(\text{Fib}) \rightarrow \text{Cat}(\text{Dbl})$$

$$\begin{array}{ccc} \text{MatCat} & \longrightarrow & \text{Fib} \\ \downarrow & \lrcorner & \downarrow \\ \text{Cat}^2 & \longrightarrow & \text{Cat} \end{array}$$

$$\text{FDC} = \text{Mnd}(\text{MatCat})$$

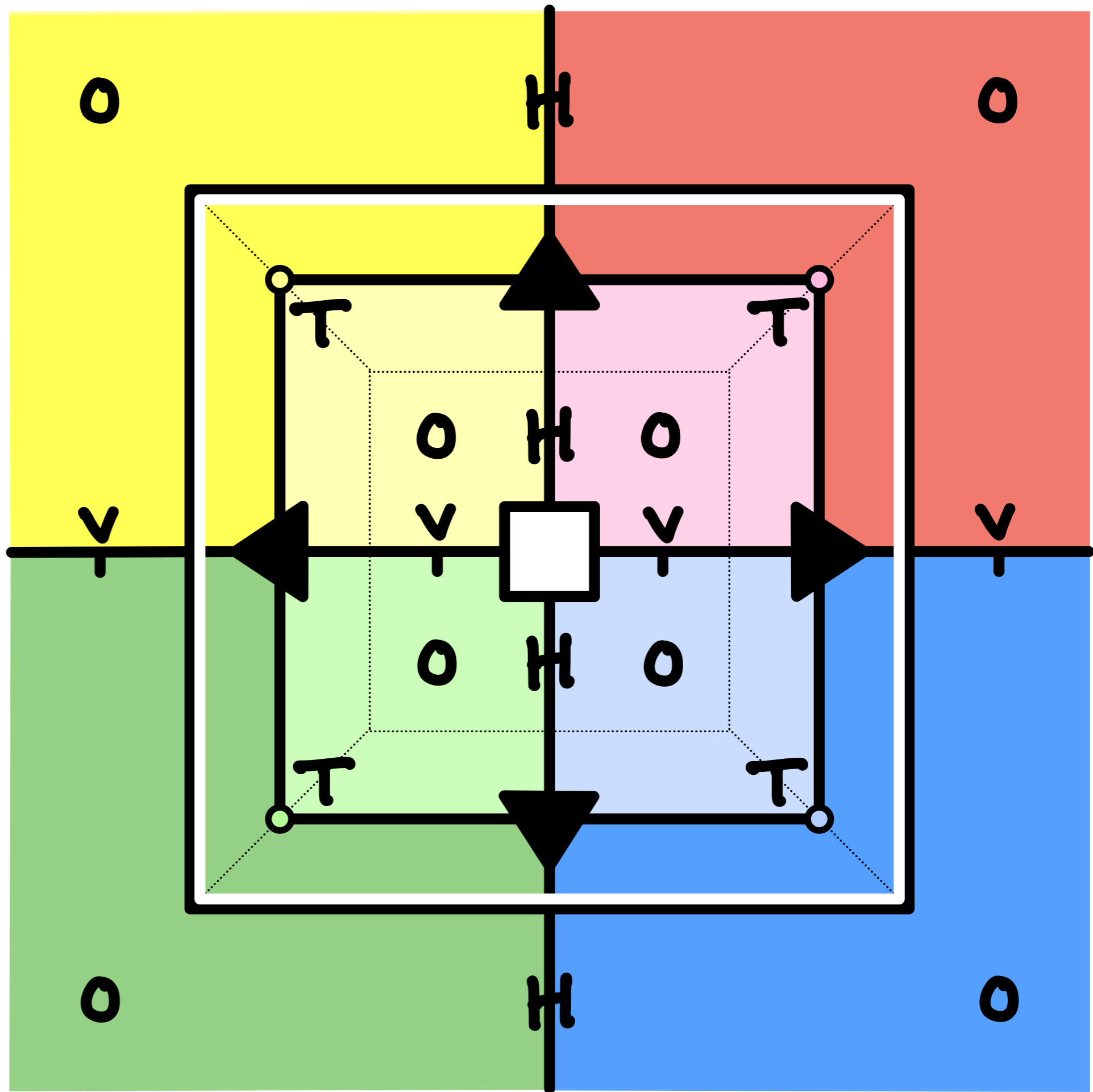
MatCat

O  
V  
H  
VH  
T  
VT  
HT  
3



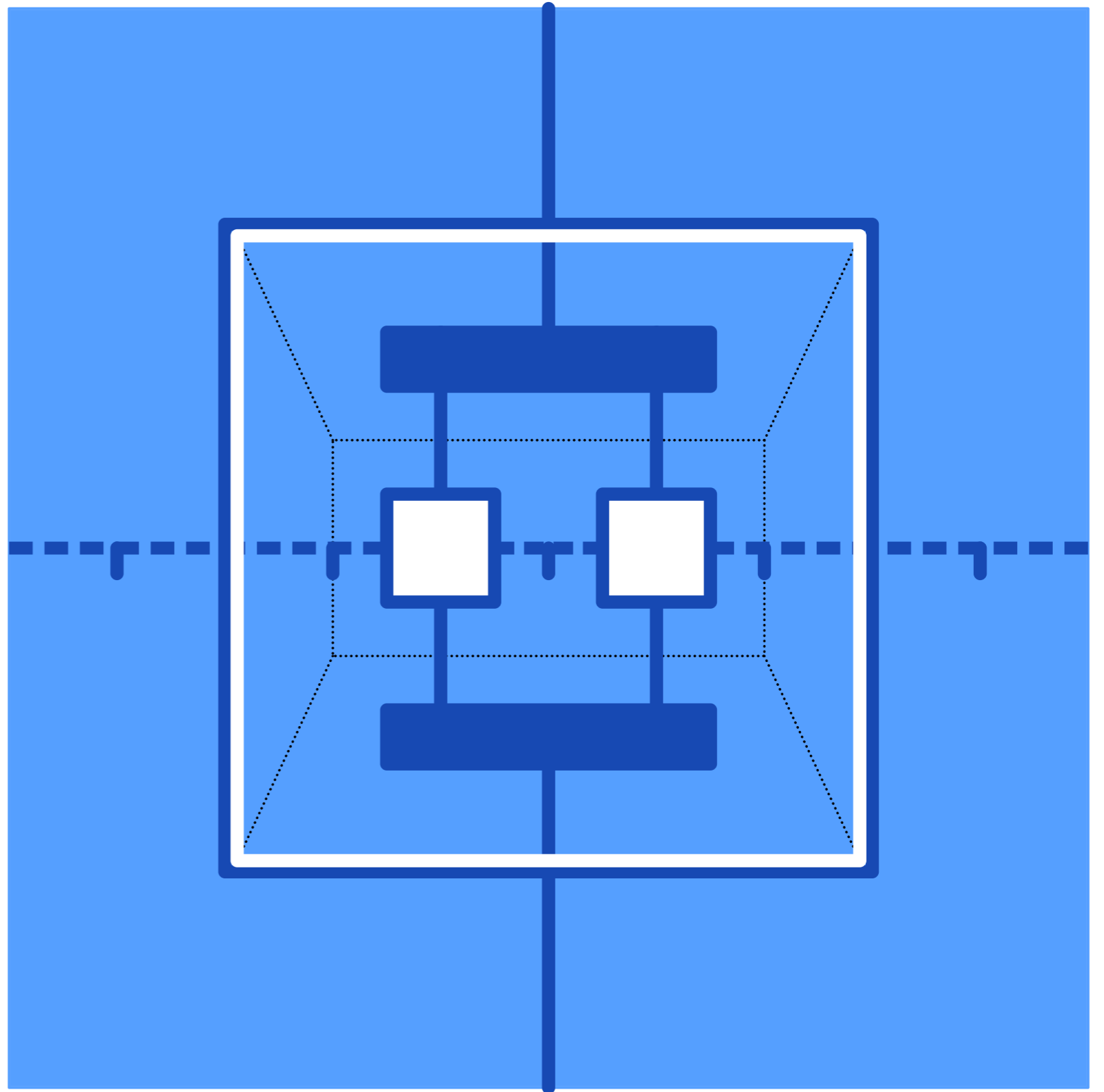
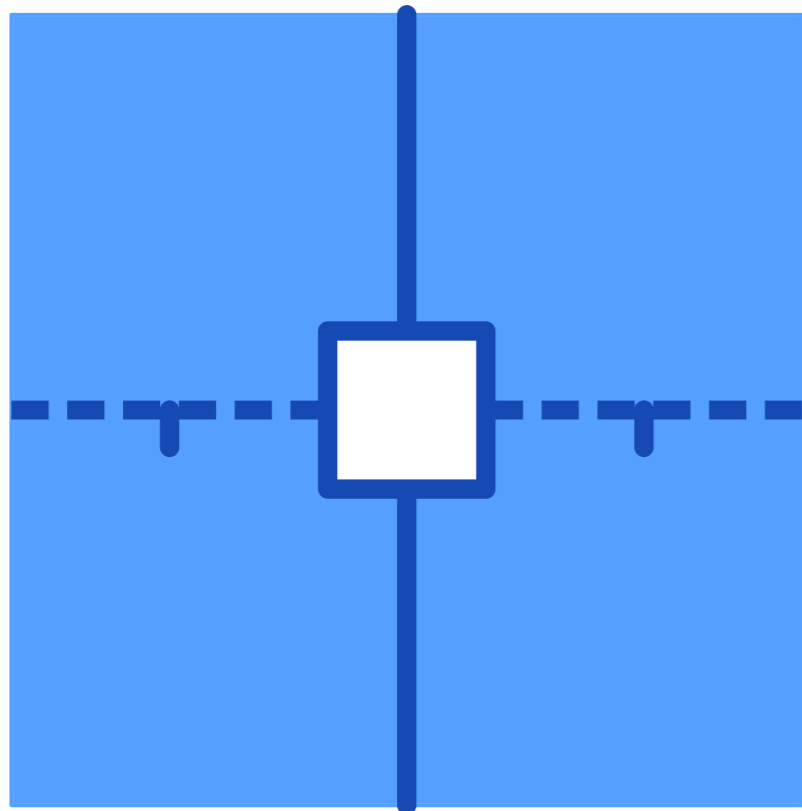
# FDC (Logos)

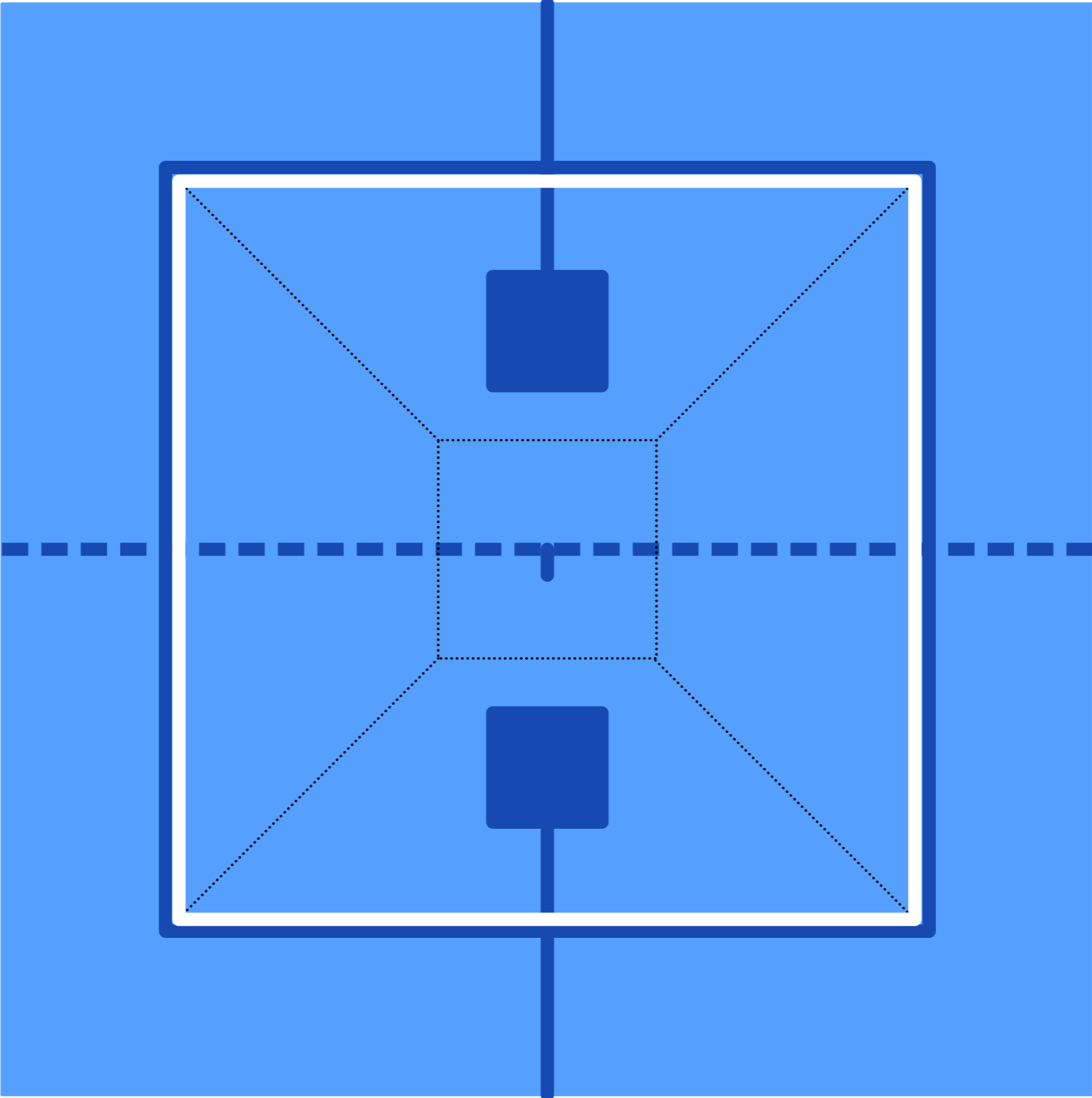
O  
 V  
 H  
 VH  
 T  
 VT  
 HT  
 3

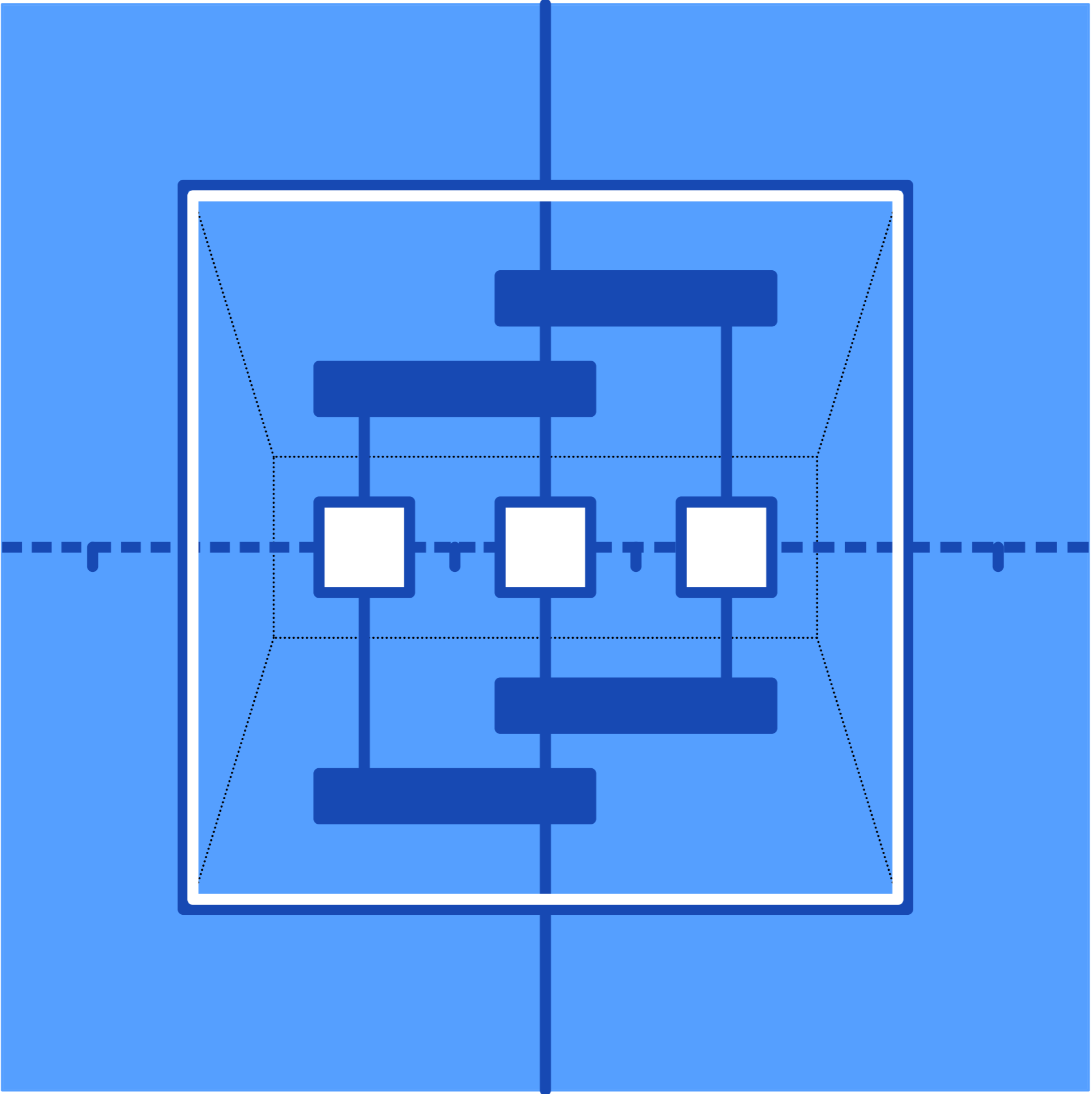




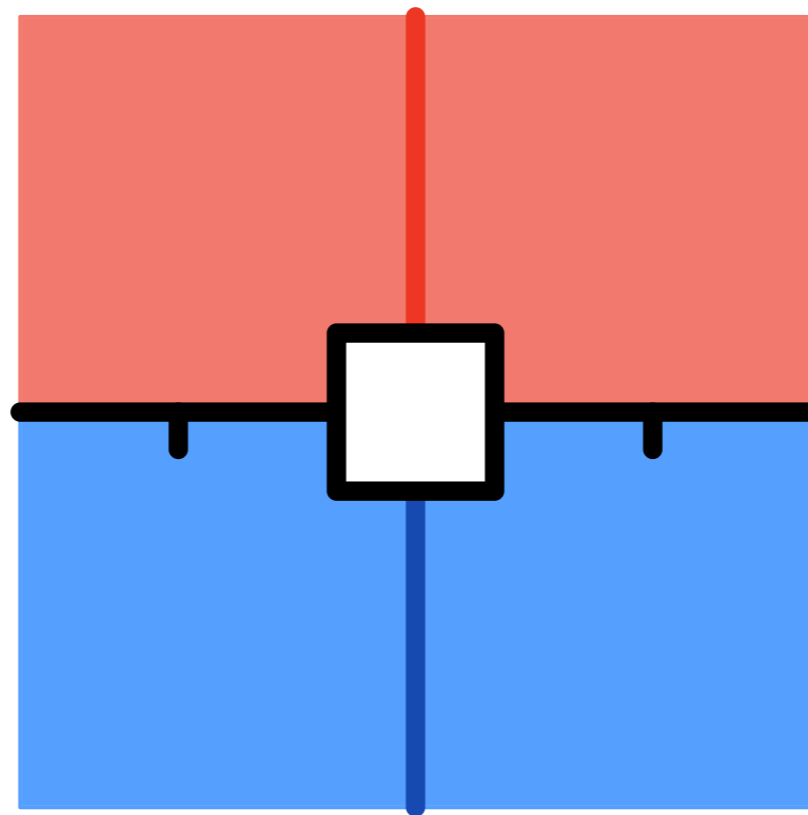
f. Double  
Category  
(Logic)

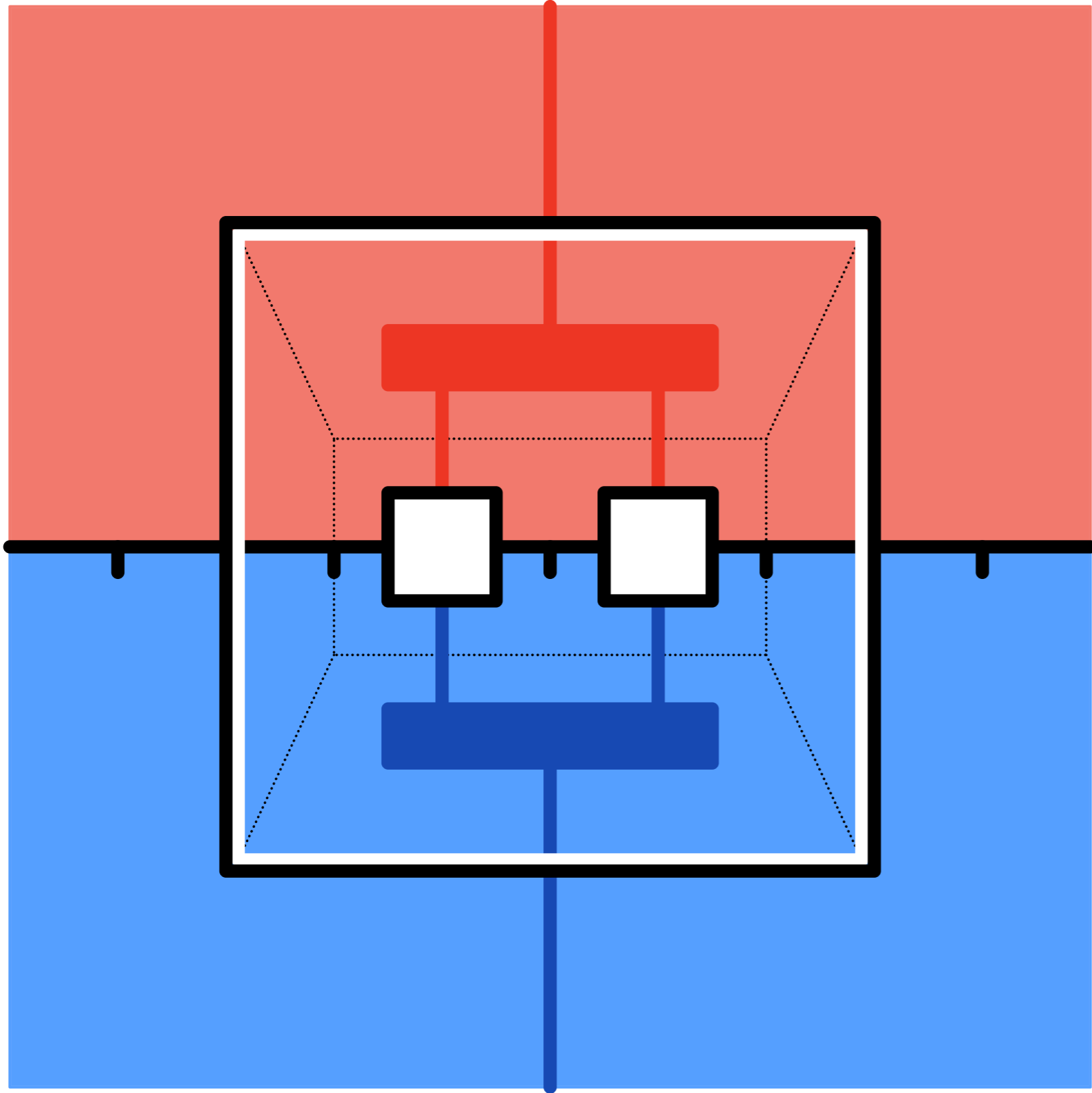




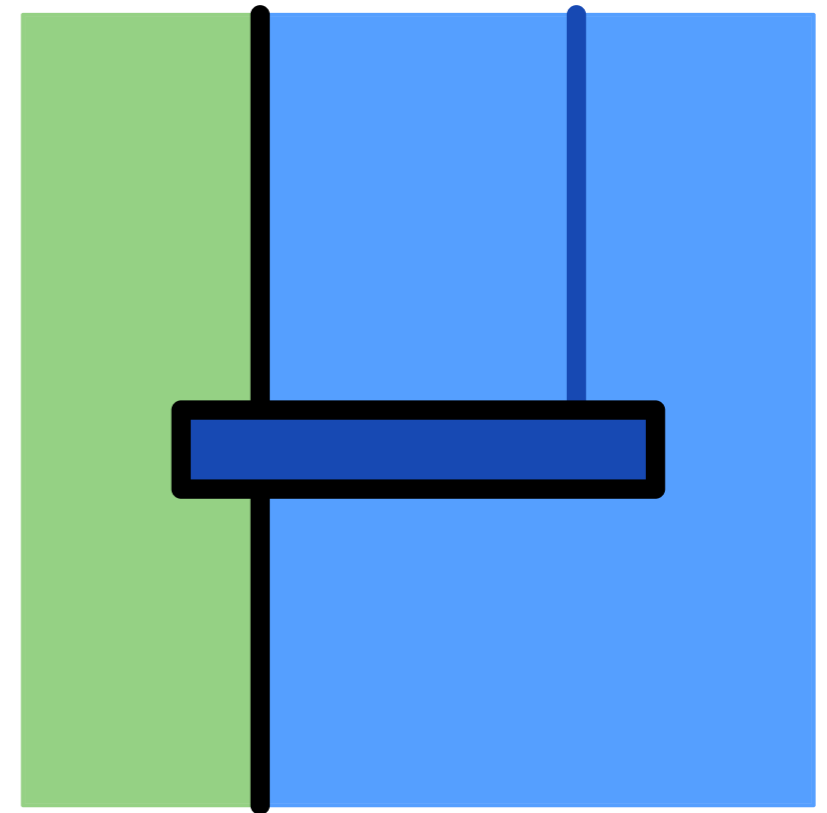
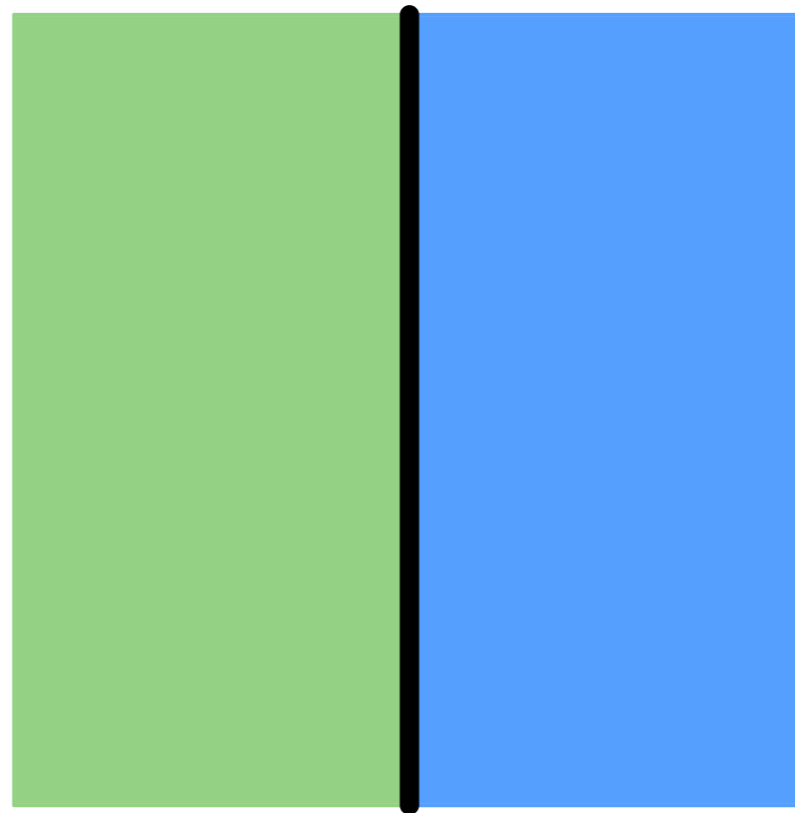
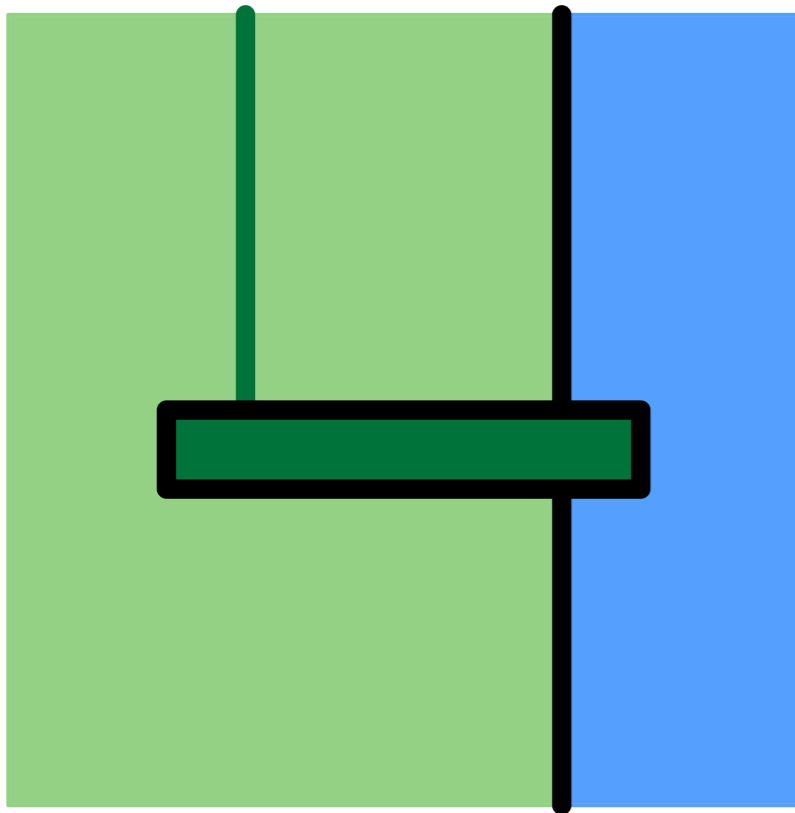


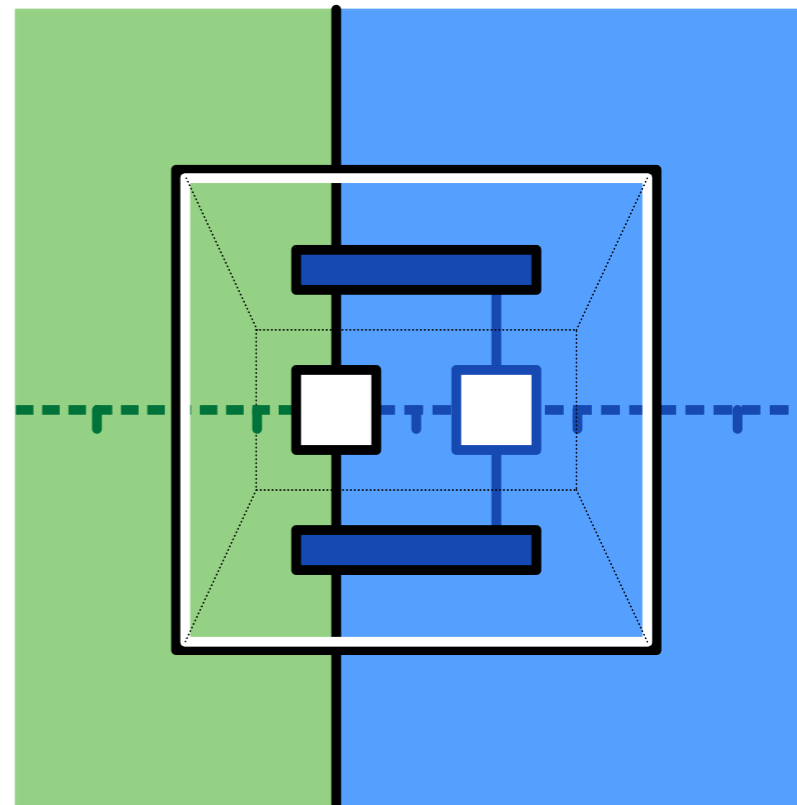
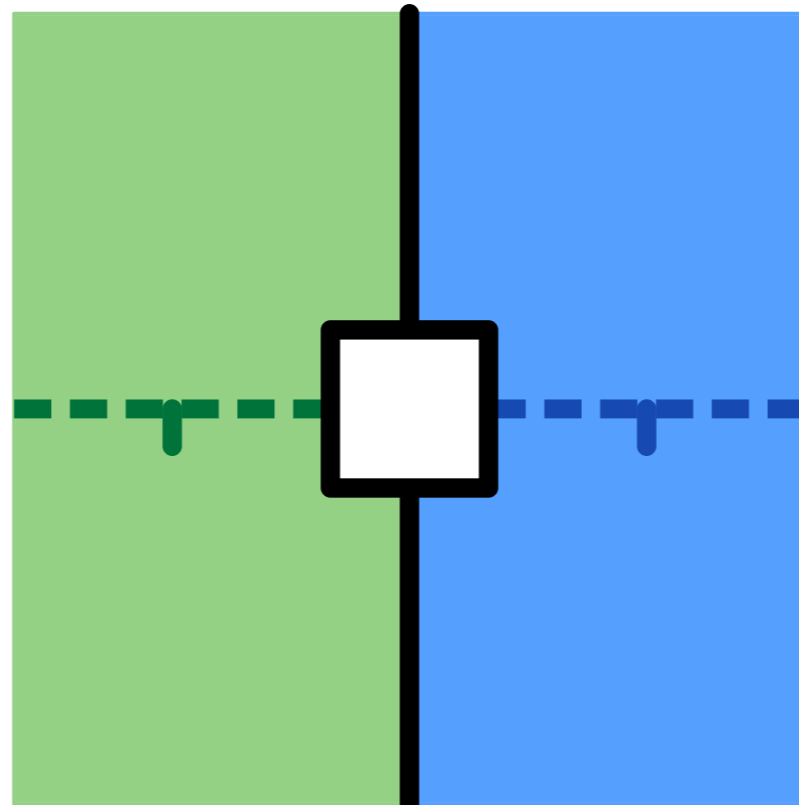
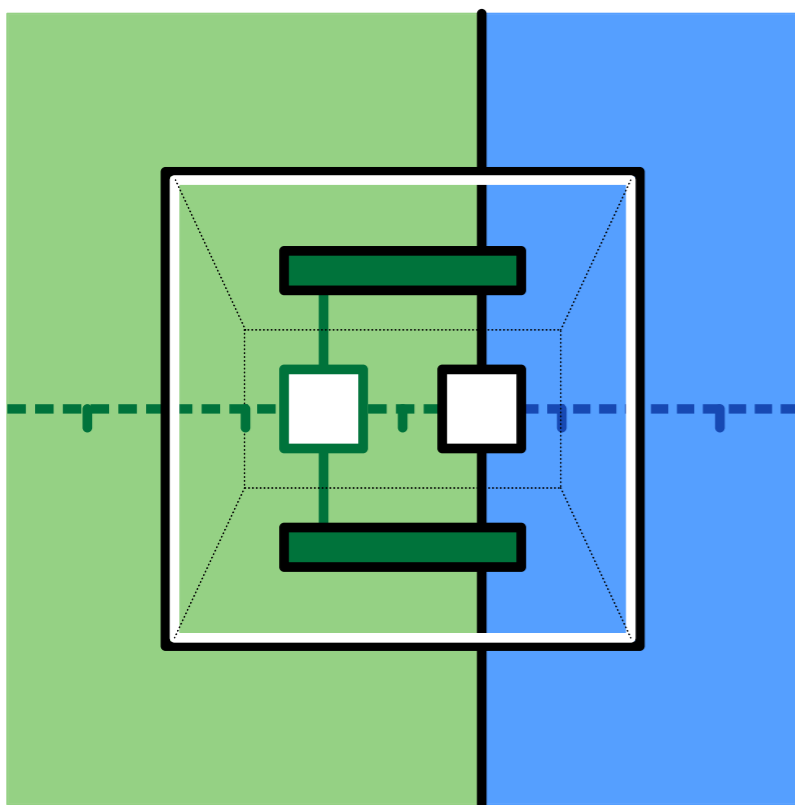
# V. Profunctor (Metaterm)



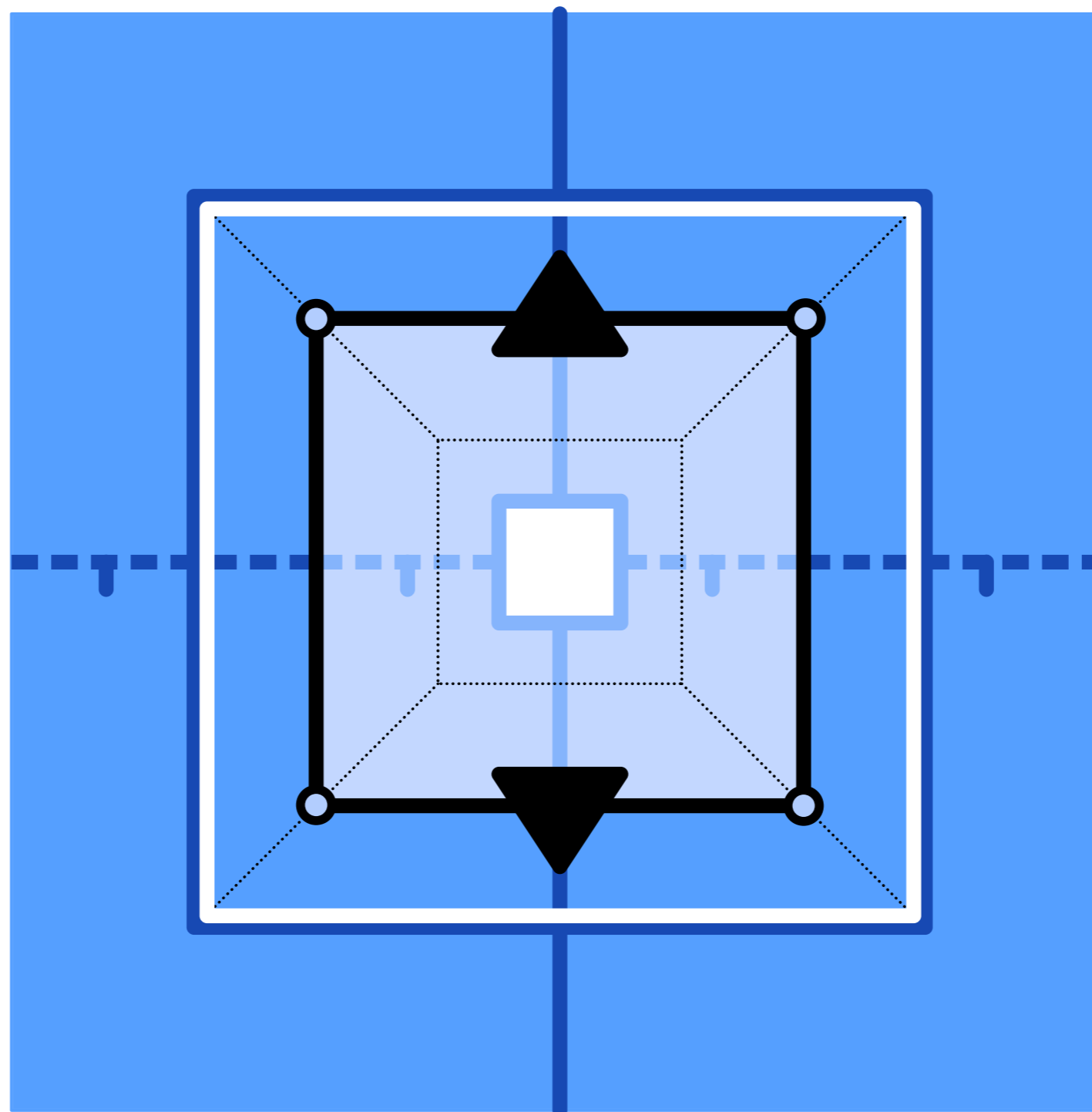


# H. Profunctor (Metajudgement)

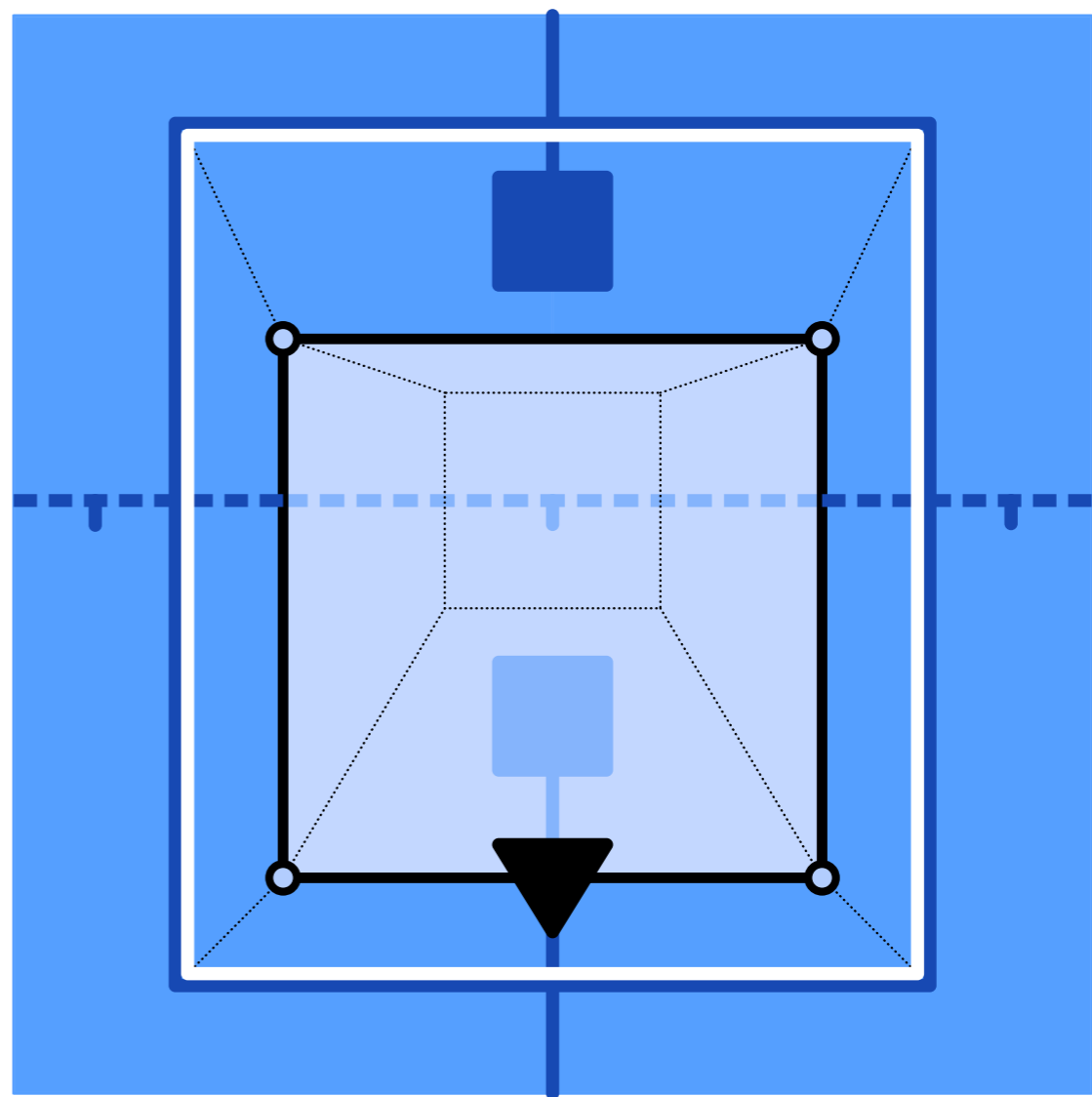
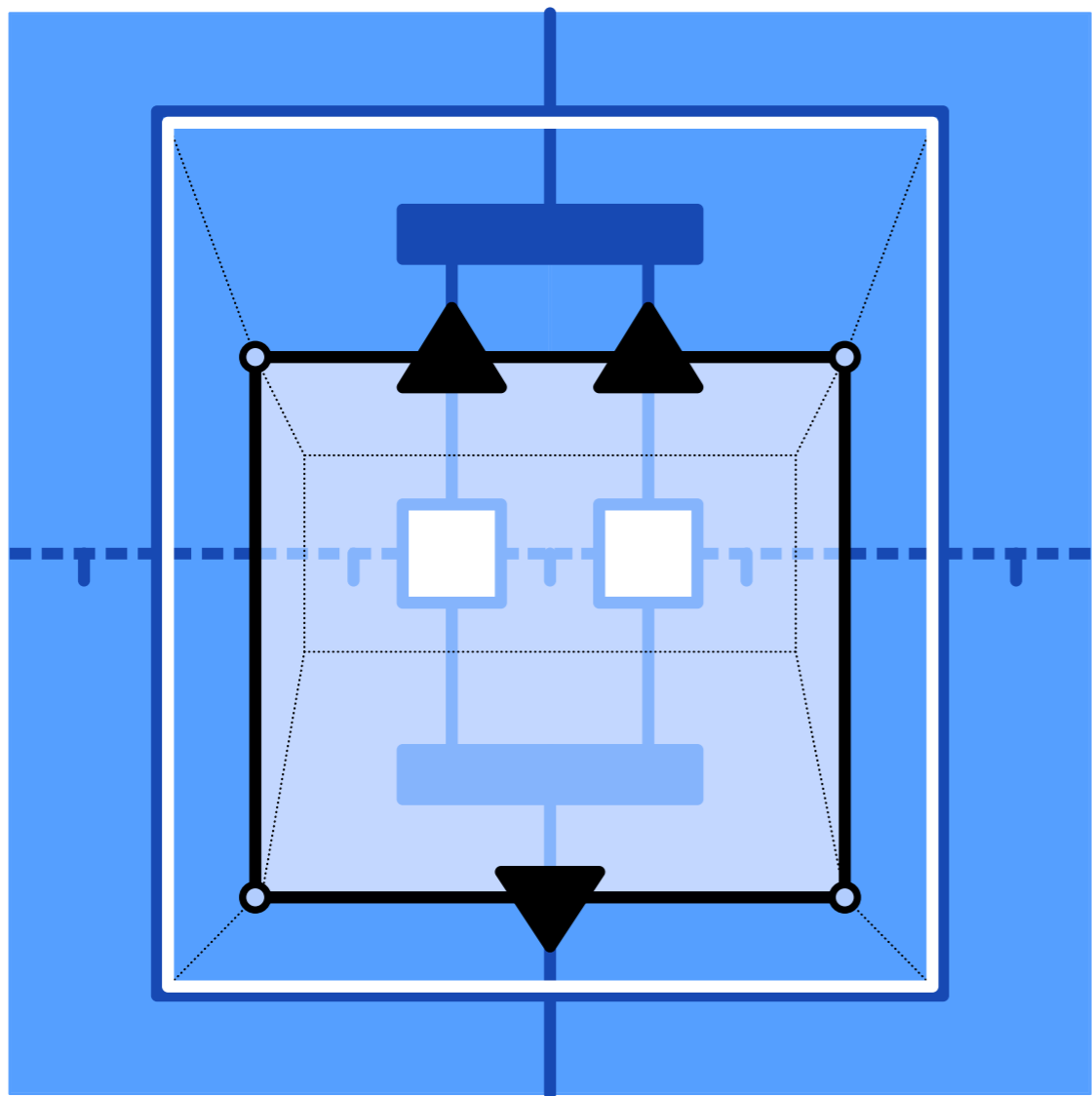


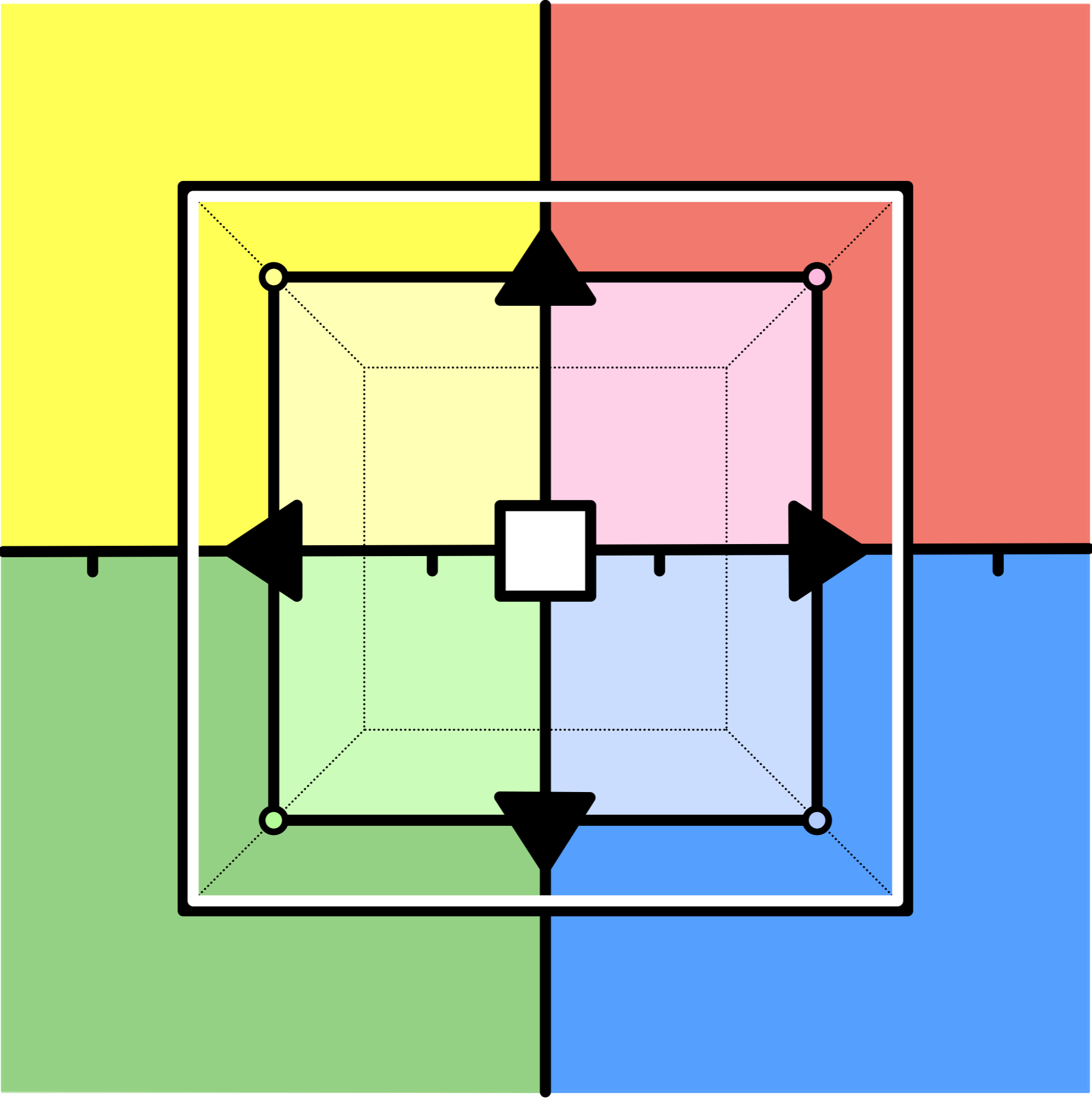


# Functor (Flow)





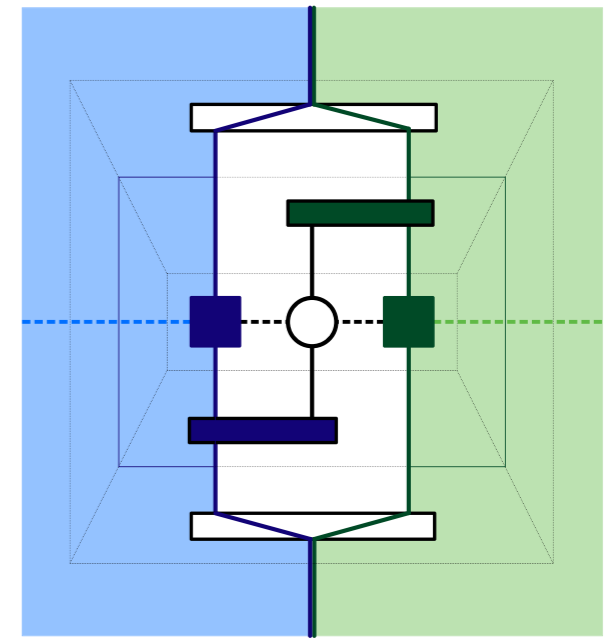




# The co/descent calculus

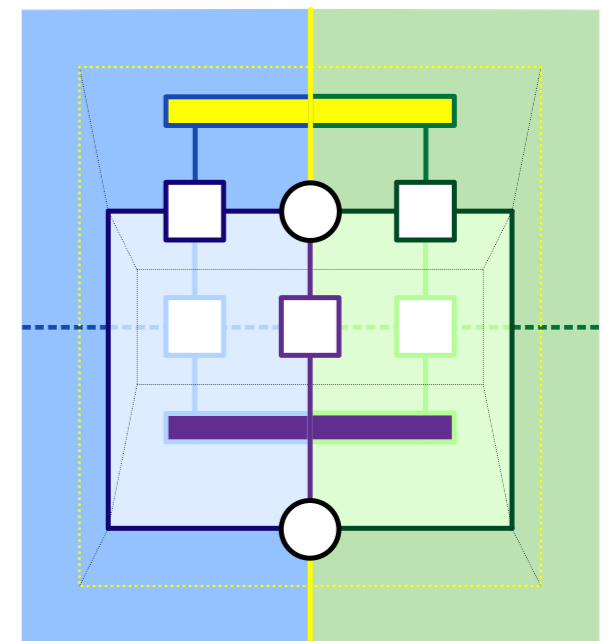
## H. Composition

Codescent :  
co inserter  
+  
co equifiers



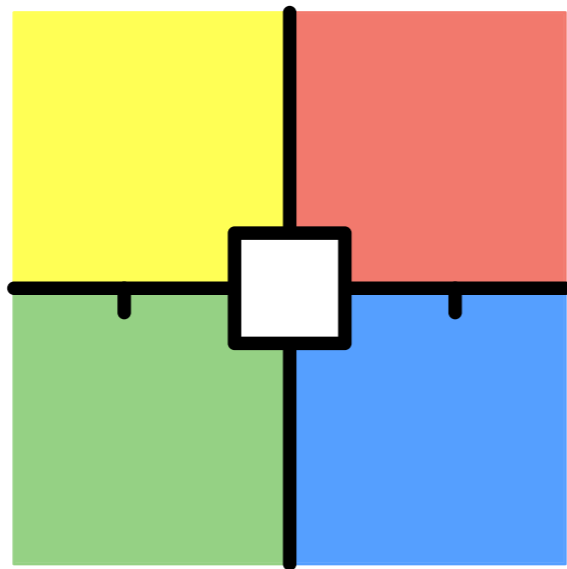
## H. Transformation

descent :  
inserter  
+  
equifiers

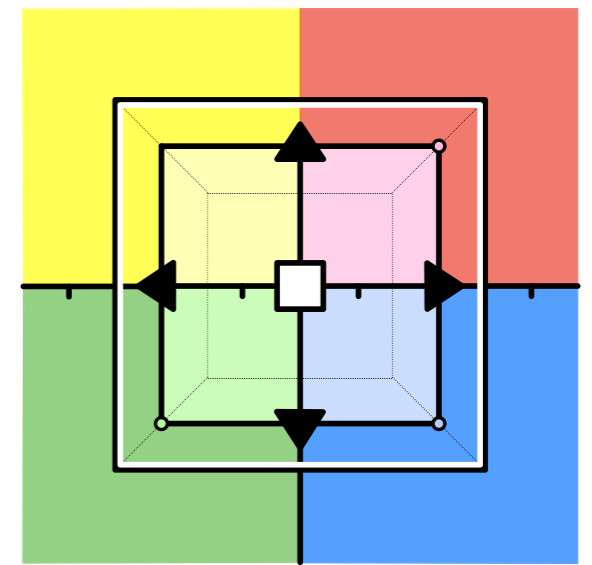


CT is a unified language.

\* a CT  
= a coend calculus  
= a logic



\* CT  
= Codescent calculus  
= Logic



\* Writing thesis  
— want to hear your thoughts!

\* Writing grants:  
a book & an app.

Thank you  
— Christian