Pno category

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Pno is a category whose objects are triples of the form (A, α, a), where

A is a set

α is a function $\alpha : A \to A$

and a is a particular, selected element.

The function α can be any function, it is not restricted in a particular way.

The arrows in this category are functions:

$f : (A, \alpha, a) \to (B, \beta, b)$

such that $f(a) = b$, and also that

$f \circ \alpha = \beta \circ f$.

For some reason, it is that last equation that I found difficult and counter-intuitive.

I believe I need more examples of such functions, to see this equation hold.

Consider some function $f : A \to B$.

What kinds of functions f are such that $f \circ \alpha = \beta \circ f$?

I can understand this part algebraically:

Assume the existence of $f : (A, \alpha, a) \to (B, \beta, b)$ and $g : (B, \beta, b) \to (C, \gamma, c)$, such that:

$f(a) = b \ g(b) = c$

and

$f \circ \alpha = \beta \circ f$

$g \circ \beta = \gamma \circ g$

Consider $g \circ f$, their composition.

$g(f(a)) = g(b) = c$

so if $h = g \circ f$, $h(a) = c$.

And $g \circ f \circ \alpha = g \circ \beta \circ f = \gamma \circ g \circ f$

So $h \circ \alpha = \gamma \circ h$

The algebra makes sense to me, I just feel like I don't intuitively understand the equation $f \circ \alpha = \beta \circ f$.

Are there some good concrete examples of functions satisfying this equation?